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ASSignment 2

1. If $y = e^{x^2+x}$ Show that $y'' = y'(2x+1) + 2y$ and hence Prove that $y^{(n+2)} = (2x+1)y^{n+1} + 2(n+1)y^n$

Solution

$y = e^{x^2+x}$ - - - - - - - - - - (1)

using chain rule; let $u = x^2 + x$ ∴ $y = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$; $\frac{du}{dx} = 2x + 1$

$\frac{dy}{du} = e^u = e^{x^2+x}$

$\frac{dy}{dx} = (2x+1)e^{x^2+x} = y$

finding y^u, using product rule

$y' = (2x+1)(e^{x^2+x})$

let $u = 2x + 1$; $du/dx = 2$

let $v = e^{x^2+x}$, $dv/dx = (2x+1)e^{x^2+x}$

$d^2y/dx^2 = y'' = u\, dv/dx + v\, du/dx$

$y'' = (2x+1)(2x+1)(e^{x^2+x}) + e^{x^2+x}(2)$

$y = e^{x^2+x}$ & $y' = (2x+1)e^{x^2+x}$

Substitute the values of y & y' in y''

$y'' = (2x+1)(2x+1)(e^{x^2+x})$

$y'' = (2x+1)y' + 2y$ - - - - - - - (ii)

To Prove $y^{(n+2)} = (2x+1)y^{n+1} + 2(n+2)y^n$ - - - - (iii)

$(2x+1)y^{n+1} + 2(n+1)y^n - y^{(n+2)} = 0$

Relating eqn (iii) and eqn (ii)

$y^{n+1} = y'$ $y^{n+2} = y''$

$= y'(2x+1)$

let $u_1 = y'(2x+1)$

$v = 2x + 1$ $\qquad y = y'$

$v' = 2$ $\qquad y^n = y^{n+1}$

$\qquad y^{n-1} = y^{(n+1)-1} = y^n$

Applying leibnitz theorem

$y^n = u^n v + n u^{(n-1)} v^{(1)} + \frac{n(n-1)}{2!} v^{(2)} u^{(n-2)} + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3 + \ldots$

$y^n = y^{(n+1)}(2x+1) + n y^n (2) + 0$

$y^n = (2x+1) y^{(n+1)} + 2(n) y^n$

let $w_2 = 2y$

$u = y \quad v = 2$

$u^n = y^n \quad v^1 = 0$

$y^n = y^n(2) + 0 = 2y^n$

let $w_3 = -y^{11}$

$y^2 - y^{11} + (y^{(2)})$

$u = y^2 \quad v = -1$

$u^n = y^{n+2} \quad v^1 = 0$

$y^n = y^{n+2}(-1) + 0 = -y^{n+2}$

$y^{n+2}(2x+1) + 2ny^n + 2y^n - y^{n+2} = 0$

$y^{n+1}(2x+1) + 2y^n(n+1) - y^{n+2} = 0$

$y^{n+1}(2x+1) + 2y^n(n+1) = y^{n+2}$

2. using the leibnitz theorem, given that (i) $y = x^3 e^{4x}$ determine y^5, ii $x^2 d^2y/dx^2 + x dy/dx + y = 0$, Show that $x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$

Solution

i $y = x^3 e^{4x}$

Using leibnitz theorem

$u = e^{4x}, u^1 = 4e^{4x}, u^{11} = 16e^{4x}, u^{111} = 64e^{4x}, u^{1v} = 256e^{4x}$

$v = x^3, v^1 = 3x^2, v^{11} = 6x, v^{111} = 0$

$\therefore u^n = 4^n e^{4x}$

$y^n = u^n v + n u^{n-1} v^1 + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 + \frac{n(n-1)(n-2)}{4!}$

$(n-3) u^{n-4} v^4 + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!} u^{n-5} v^5$

$= 4^n e^{4x}(x^3) e^{4x} + n 4^{n-1} e^{4x}(3x^2) + \frac{n(n-1)}{2!} 4^{n-2} e^{4x} 6x + (n)(n-1)$

$u^{n-5} v^5 \dots$

$= 4^n e^{4x}(x^3) e^{4x} + n 4^{n-1} e^{4x}(3x^2) + \frac{n(n-1) 4^{n-2}}{2!} e^{4x}\, 6x + \frac{(n)(n-1)(n-2)}{3!} 4^{n-3} e^{4x}\, 6$

$\frac{n(n-1)(n-2)(n-3)}{4!} 4^{n-4}(e^{4x})(0)$

Since $n=5$

$y^5 = 4^5 e^{4x} x^3 e^{4x} + 5(4^4 e^{4x}) 3x^2 + \frac{5(5-1)}{2!} 4^3 e^{4x}\, 6x + \frac{5(5-1)(5-2)}{3!} 4^2 e^{4x}\, 6 + 0$

$y^5 = 4^5 e^{4x} x^3 + 5(4^4 e^{4x}) 3x^2 + \frac{5(4)}{2!} 4^3 e^{4x}\, 6x + \frac{5(4)(3)}{3!} 4^2 e^{4x}\, 6 + 0$

$y^5 = 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4x} x + 960 e^{4x}$

$\underline{y^5 = 64 e^{4x}(16x^3 + 60x^2 + 60x + 15)}$

$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$

$x^2 y'' + x y' + y = 0$

Let $g_1 = x^2 y''$

$V = x^2 \qquad u^n = y'' = y^{(n+2)}$

$V' = 2x \qquad u^n = y^{n+2}$

$V'' = 2 \qquad u^{n-1} = y^{(n+2)-1} = y^{n+1}$

$V''' = 0 \qquad u^{n-2} = y^{(n+2)-2} = y^n$

Using Leibnitz Theorem

$g_1^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + 0$

$g_1^n = y^{(n+2)} x^2 + n y^{n+1}(2x) + \frac{n(n-1)}{2!} y^n (2) + 0$

$g_1^n = x^2 y^{n+2} + (2x)(n) y^{n+1} + n(n-1) y^n$

$g_1^n = x^2 y^{n+2} + 2nx\, y^{n-1} + n^2 - n\, y^n$

Let $g_2 = x y'$

$u = y', \; u^n = y^{(n+1)}$

$v' = 1$ $\quad u^{n+1} = y^{n+1-1} = y^n$

$v^2 = 0$ $\quad u^{n-2} = y^{(n+1)-2} = y^{n-1}$

$g_2^n = u^n v + n u^{n-1} v' + 0$

$g_2^n = y^{(n+1)} x + n(y^n)(1)$

$g_2^n = x y^{n+1} + n y^n$

Let $g_3 = y$

$v = 1$ $\quad u = y,\ u^n = y^n$

$v' = 0$ $\quad y^{n-1} = y^{n-1}$

$g_3^n = u^n v + 0$

$\quad\quad = y^n(1)$

$g_3^n = y^n$

$g_1^n + g_2^n + g_3^n = 0$

$x^2 y^{n+2} + 2nx y^{n+1} + x y^{n+1} + (n^2 - n) y^n + n y^n + y^n = 0$

$\therefore \underline{x^2 y^{n+2} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0}$