21-10-19

ENG 381
Assignment III

1. Solution

(a) $x(x-1)y'' + (3x-1)y' + y = 0$

for the nth derivative,

$$x(x-1)y^{n+2} + (2x-1)ny^{n+1} + n(n-1)y^n + (3x-1)y^{n+1} + 3ny^n + y^n = 0$$

$$x(x-1)y^{n+2} + y^{n+1}(2xn - n + 3x - 1) + y^n(n^2 - n + 3n + 1) = 0$$

at $x = 0$,

$$(y^{n+1})_0(-n-1) + (y^n)_0(n^2 + 2n + 1) = 0$$

$$-(n+1)(y^{n+1})_0 = -(n^2 + 2n + 1)(y^n)_0$$

$$(n+1)(y^{n+1})_0 = (n^2 + 2n + 1)(y^n)_0$$

$$(y^{n+1})_0 = (n+1)(y^n)_0$$

at $n = 0$,

$$(y^{0+1})_0 = (0+1)(y^0)_0$$

$$(y^1)_0 = (y)_0$$

at $n = 1$,

$$(y^{1+1})_0 = (1+1)(y^1)_0$$

$$(y^2)_0 = 2(y^1)_0$$

at $n = 2$,

$$(y^{2+1})_0 = (2+1)(y^2)_0$$

$$(y^3)_0 = 3(y^2)_0$$

$$(y^3)_0 = 6(y^1)_0$$

at $n = 3$,

$(y^3)_0 = (3+1)(y^4)_0$

$(y^4)_0 = 4(y^3)_0 = 4 \times 6(y^2)_0 = 24(y^1)_0$

at $n = 4$,

$(y^5)_0 = (4+1)(y^5)_0$

$= 5(y^4)_0$

$= 5 \times 24(y^1)_0$

$(y^5)_0 = 120(y^1)_0$

at $n = 5$,

$(y^6)_0 = (5+1)(y^6)_0$

$= 6(y^5)_0$

$= 6 \times 120(y^1)_0$

$(y^6)_0 = 720(y^1)_0$

at $n = 6$,

$(y^7)_0 = (6+1)(y^6)_0$

$= 7(y^6)_0$

$= 7 \times 720(y^1)_0$

$(y^7)_0 = 5040(y^1)_0$

from maclaurin series,

$$y = (y)_0 + x(y^1)_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \dots$$

$$\therefore y = (y)_0 + x(y^1)_0 + \frac{x^2}{2!}(2(y^1)_0) + \frac{x^3}{3!}(6(y^1)_0) + \frac{x^4}{4!}(24(y^1)_0)$$

$$+ \frac{x^5}{5!}(120(y^1)_0) + \frac{x^6}{6!}(720(y^1)_0) + \frac{x^7}{7!}(5040(y^1)_0) + \dots$$

$y = (y)_0 + x(y')_0 + x^2(y')_0 + x^3(y')_0 + x^4(y')_0 + x^5(y')_0 + x^6(y')_0 + x^7(y')_0 + \ldots$

$y = (y)_0 + (y')_0 [x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \ldots]$

but $(y)_0 = 0.0005m$ and $(y')_0 = 0.0005$,

$\therefore y = 0.0005 + 0.0005 [x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7]$

(b) When $x = 5m$,

$y = 0.0005 + 0.0005 [5 + 25 + 125 + 625 + 3125 + 15625 + 78125]$

$y = 0.0005 + 0.0005 [97655]$

$\therefore y = 48.828m$

When $x = 8m$,

$y = 0.0005 + 0.0005 [8 + 64 + 512 + 4096 + 32768 + 262144 + 2097152$

$y = 0.0005 + 0.0005 [2396744]$

$\therefore y = 1198.3725m$

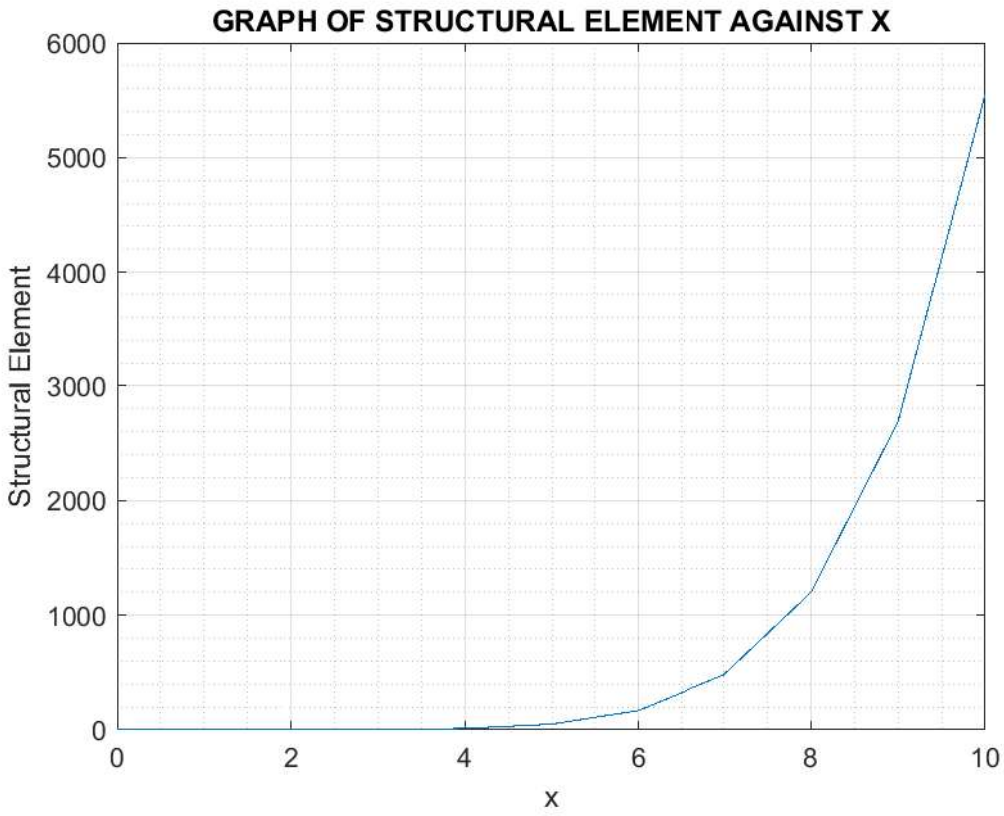
When $x = 10m$,

(c) $y = 0.0005 + 0.0005 [10 + 100 + 1000 + 10000 + 100000 + 1000000 + 10000000]$

$y = 0.0005 + 0.0005 [11111110]$

$y = 5555.5555m$

```
commandwindow
clear
clc
syms x
syms y
x= (0:10);
y= 0.0005+0.0005*(x+(x.^2)+(x.^3)+(x.^4)+(x.^5)+(x.^6)+(x.^7));
plot(x,y)
grid on
grid minor
xlabel('x')
ylabel('Structural Element')
```


GRAPH OF STRUCTURAL ELEMENT AGAINST X
Structural Element
x
0
1000
2000
3000
4000
5000
6000
0
2
4
6
8
10