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Question 1

Find the area bounded by the curves $y = 3e^{2x}$ and $y = 3e^{-x}$ and the ordinates at $x = 1$ and $x = 2$

Solution

For curve $y = 3e^{2x}$

$$A_1 = \int_1^2 y\,dx = \int_1^2 3e^{2x}\,dx$$

$$A_1 = \left[\frac{3}{2}e^{2x}\right]_1^2$$

$$A_1 = \frac{3}{2}\left[e^{2x}\right]_1^2 = \frac{3}{2}\left[e^{2\times 2} - e^{2\times 1}\right]$$

$$= \frac{3}{2}\left[e^4 - e^2\right] = \frac{3}{2}\left[47.2091\right]$$

$$A_1 = 70.8137 \text{ unit}^2$$

For curve $y = 3e^{-x}$

$$A_2 = \int_1^2 y\,dx = \int_1^2 3e^{-x}\,dx$$

$$= 3\int_1^2 e^{-x}\,dx = -3\left[e^{-x}\right]_1^2$$

$$= -3\left[e^{-2} - e^{-1}\right] = -3\left[-0.2325\right]$$

$$A_2 = 0.6975 \text{ unit}^2$$

$\therefore$ The Area bounded by the curves $y = 3e^{2x}$ and $y = 3e^{-x}$ is

$$A = A_1 - A_2$$

$$A = \left[70.8137 - 0.6975\right] \text{unit}^2$$

$$A = 70.1162 \text{ unit}^2$$

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Question 2:

The parametric equations of a curve are $y = 2\sin\frac{\pi}{10}t$ and $x = 2 + 2t - 2\cos\frac{\pi}{10}t$. find the area under the curve between $t = 0$ and $t = 10$

Solution

$$A = \int_0^{10} y\,dx = \int_0^{10} 2\sin\frac{\pi}{10}t\,dx$$

Since $x = 2 + 2t - 2\cos\frac{\pi}{10}t$

then

$$\frac{dx}{dt} = 2 + \frac{2\pi}{10}\sin\frac{\pi}{10}t$$

$$\frac{dx}{dt} = 2 + \frac{\pi}{5}\sin\frac{\pi}{10}t$$

$$\therefore\ dx = \left(2 + \frac{\pi}{5}\sin\frac{\pi}{10}t\right)dt$$

therefore $A = \displaystyle\int_0^{10}\left(2\sin\frac{\pi}{10}t\right)\left(2 + \frac{\pi}{5}\sin\frac{\pi}{10}t\right)dt$

$$= \int_0^{10}\left(4\sin\frac{\pi}{10}t + \frac{2\pi}{5}\sin^2\frac{\pi}{10}t\right)dt$$

$$\cancel{= -\frac{40}{\pi}\cos\frac{\pi}{10}t} \quad \int_0^{10}\left(4\sin\frac{\pi}{10}t\right)dt + \int_0^{10}\left(\frac{2\pi}{5}\sin^2\frac{\pi}{10}t\right)dt$$

Integrating $\displaystyle\int_0^{10}\left(4\sin\frac{\pi}{10}t\right)dt$ we have

$$\left[-\frac{40}{\pi}\cos\frac{\pi}{10}t\right]_0^{10}$$

Integrating $\displaystyle\int\left(\frac{2\pi}{5}\sin^2\frac{\pi}{10}t\right)dt$

from trignometry ; $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

therefore $\sin^2\frac{\pi}{10}t = \frac{1}{2}\left(1 - \cos\frac{\pi}{5}t\right)$

then integrating we have

$$\frac{2\pi}{5}\int\left(\sin^2\frac{\pi}{10}t\right)dt$$

$$\frac{2\pi}{5}\int\left(\frac{1}{2}\left(1-\cos\frac{\pi}{5}t\right)\right)dt$$

$$\frac{2\pi}{10}\int 1-\cos\frac{\pi}{5}t \; dt = \frac{2\pi}{10}\left[t-\frac{5}{\pi}\sin\frac{\pi}{5}t\right]$$

$$= \frac{\pi}{5}\left[t-\frac{5}{\pi}\sin\frac{\pi}{5}t\right]_0^{10}$$

therefore

$$\int_0^{10}\left(4\sin\frac{\pi}{10}t + \frac{2\pi}{5}\sin^2\frac{\pi}{10}t\right)dt =$$

$$\left[-\frac{40}{\pi}\cos\frac{\pi}{10}t + \frac{\pi}{5}t - \sin\frac{\pi}{5}t\right]_0^{10}$$

$$\left[-\frac{40}{\pi}\cos\frac{\pi}{10}(10) + \frac{\pi}{5}(10) - \sin\frac{\pi}{5}(10)\right] - \left[-\frac{40}{\pi}\cos\frac{\pi}{10}(0) + \frac{\pi}{5}(0) - \sin\frac{\pi}{5}(0)\right]$$

$\pi = 180$ ~~for~~ angles then we have

$$\left[\frac{40}{\pi} + 2\pi - 0\right] - \left[-\frac{40}{\pi} + 0 - 0\right]$$

$$\frac{40}{\pi} + 2\pi + \frac{40}{\pi} = \left(\frac{80}{\pi} + 2\pi\right)\text{unit}^2$$

therefore $A = 25.4648 + 6.2832$

$A = 31.748 \text{ unit}^2$