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ENG 381 ASSIGNMENT 3

The model for deformation (y) of a structural element is represented by the expression given in equation (1)

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Given that $y(0) = 0.0005m$ and $y'(0) = 0.0005$, applying Leibnitz - Maclaurin method

a) Obtain the power series solution of the model up to and including the terms in x^2
b) estimate the appropriate deformation when $x = 5, 8$ and $10m$, and
c) with the aid of a MATLAB mfile program, plot the response of the structural element for $0 \leq x \leq 10m$

Solution

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Expanding the bracket

$$(x^2-x)y'' + (3x-1)y' + y = 0$$

$$w_1 = (x^2-x)y''$$

$$w_2 = (3x-1)y'$$

$$w_3 = y$$

Using Leibnitz theorem

for w_1

$$U^n V + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V^2 + \frac{n(n-1)(n-2)}{3!}U^{n-3}V^3 + \ldots$$

for w_1		for w_2		for w_3	
$U = y^2$	$V = x^2 - x$	$U = y'$	$V = 3x-1$	$U = y$	$V = 1$
$U^n = y^{n+2}$	$V' = 2x-1$	$U^n = y^{n+1}$	$V' = 3$	$U^n = y^n$	$V' = 0$
$U^{n-1} = y^{n+1}$	$V^2 = 2$	$U^{n-1} = y^n$	$V^2 = 0$		
$U^{n-2} = y^n$	$V^3 = 0$				

$$W_1 = y^{(n+2)}.(x^2-x) + n.y^{(n+1)}(2x\cancel{\pm}1) + \frac{n(n-1)}{\cancel{2}}.y^n.\cancel{2}$$

$$W_2 = y^{(n+1)}.(3x-1) + 3n y^n$$

$$W_3 = y^n$$

Summing all together.

$$y^{(n+2)}.(x^2-x) + (2nx\cancel{\pm n})y^{(n+1)} + y^n(n^2-n) + y^{(n+1)}(3x-1) + 3ny^n + y^n = 0$$

assuming $x=0$

$$y^{(n+2)}.\cancel{(0^2-0)} + (2n(0)\cancel{-n})y^{(n+1)} + y^n(n^2-n) + y^{(n+1)}(3(0)-1) + 3n y^n + y^n = 0$$

$$= -n y^{(n+1)} + y^n(n^2-n) - y^{(n+1)} + 3n y^n + y^n = 0$$

collecting liketerms

$$= y^{(n+1)}(-n-1) + y^n(n^2-n+3n+1) = 0$$

$$= y^{(n+1)}(-n-1) + y^n(n^2+2n+1) = 0$$

$$= -y^{n+1}(n+1) + y^n(n^2+2n+1) = 0$$

$$= y^{(n+1)}(n+1) = y^n \underbrace{(n^2+2n+1)}_{(n+1)(n+1)}$$

$$\therefore y^{(n+1)}(n+1) = y^n(n+1)(n+1)$$

Divide both side by $(n+1)$

$$\frac{y^{(n+1)}\cancel{(n+1)}}{\cancel{(n+1)}} = \frac{y^n(n+1)\cancel{(n+1)}}{\cancel{(n+1)}}$$

$$\boxed{y^{(n+1)} = y^n(n+1)}$$ — Recurrence relation.

$$\left(y^{n+1}\right)_0 = y^n(n+1).$$

$$(y^0)_0 = 0.0005$$

$$(y^1)_0 = 0.0005$$

when $n=0$

$$\left[y^{(0+1)}\right]_0 = (0+1)\left(y^0\right)_0$$

$$\left[y^{(1)}\right]_0 = 1\left[y^0\right]_0$$

When $n=1$

$$\left[y^{(1+1)}\right]_0 = (1+1)\left(y^1\right)_0$$

$$\left[y^{(2)}\right]_0 = 2\left(y^1\right)_0$$

when $n=2$

$$\left(y^3\right)_0 = (2+1)y^2$$

$$\left(y^3\right)_0 = 3\left[y^2\right]_0 = 3(2)\left[y^{(1)}\right]_0$$

$$\left(y^{(3)}\right)_0 = 6\left(y^1\right)_0$$

When $n=3$

$$\left(y^4\right)_0 = (3+1)y^3$$

$$\left(y^4\right)_0 = 4\left[y^3\right]_0 = 4\left[6\left(y^1\right)_0\right] = 24\left(y^1\right)_0$$

when $n=4$

$$\left(y^5\right)_0 = (4+1)y^4$$

$$\left(y^5\right)_0 = 5\left(y^4\right) = 5\left[24\left(y^1\right)_0\right] = 120\left(y^1\right)_0$$

when $n=5$

$$y^6 = (5+1)y^5$$

$$\left(y^6\right)_0 = 6\left(y^5\right) = 6\left[120\left(y^1\right)_0\right] = 720\left(y^1\right)_0$$

When $n=6$

$$y^7 = (6+1)y^6$$

$$\left(y^7\right) = 7\left(y^6\right) = 7\left(720\left(y^1\right)_0\right) = 5040\left(y^1\right)_0$$

$\therefore y_n =$

Using Maclaurin Series:

$$Y = (Y^0)_0 + x(Y^1)_0 + \frac{x^2}{2!}(Y^2)_0 + \frac{x^3}{3!}(Y^3)_0 + \frac{x^4}{4!}(Y^4)_0 + \frac{x^5}{5!}(Y^5) + \frac{x^6}{6!}(Y^6) + \frac{x^7}{7!}(Y^7)_0$$

$$Y = (Y^0)_0 + x(Y^0)_0 + \frac{x^2}{\cancel{2!}}(\cancel{2}Y^1)_0 + \frac{x^3}{\cancel{3!}}(\cancel{6}Y^1)_0 + \frac{x^4}{\cancel{4!}}(\cancel{24}Y^1) + \frac{x^5}{\cancel{5!}}(\cancel{120}Y^1)$$
$$+ \frac{x^6}{\cancel{6!}}(\cancel{720}Y^1) + \frac{x^7}{\cancel{7!}}(\cancel{5040}Y^1)$$

$$Y = Y^0(1+x) + (x^2+x^3+x^4+x^5+x^6+x^7)Y^1$$

$$\therefore Y = 0.0005(1+x) + (x^2+x^3+x^4+x^5+x^6+x^7)\,0.0005$$

ii. Estimate the approximate deformation when $x = 5, 8$ and $10\,m$

when $x = 5\,m$

$$Y = Y_0(1+5) + (5^2+5^3+5^4+5^5+5^6+5^7)\,0.0005$$
$$Y = 0.0005(1+5) + (25+125+625+3125+15625+78125)\,0.0005$$
$$Y = 48.828\,m$$

when $x = 8\,m$

$$Y = Y_0(1+8) + (8^2+8^3+8^4+8^5+8^6+8^7)\,0.0005$$
$$Y = 0.0005(1+8) + (64+512+4096+32768+262144+2097152)\,0.0005$$
$$Y = 1198.3725\,m$$

when $x = 10\,m$

$$Y = Y_0(1+10) + (10^2+10^3+10^4+10^5+10^6+10^7)\,Y_1$$
$$Y = 0.0005(1+10) + (100+1000+10000+100000+1000000+10000000)\,0.0005$$
$$Y = 5555.56\,m$$

MATLAB mfile

Command window

Clear

clc

Close all

x = 0:0.01:10

a) y = (0.0005*(1+x)) + ((x.^2+x.^3+x.^4+x.^5+x.^6+x.^7)*0.0005)

b) yn = subs(y)

c) = plot (x,yn)

xlabel('m')

ylabel('Deflection')

axis tight

grid on

grid minor

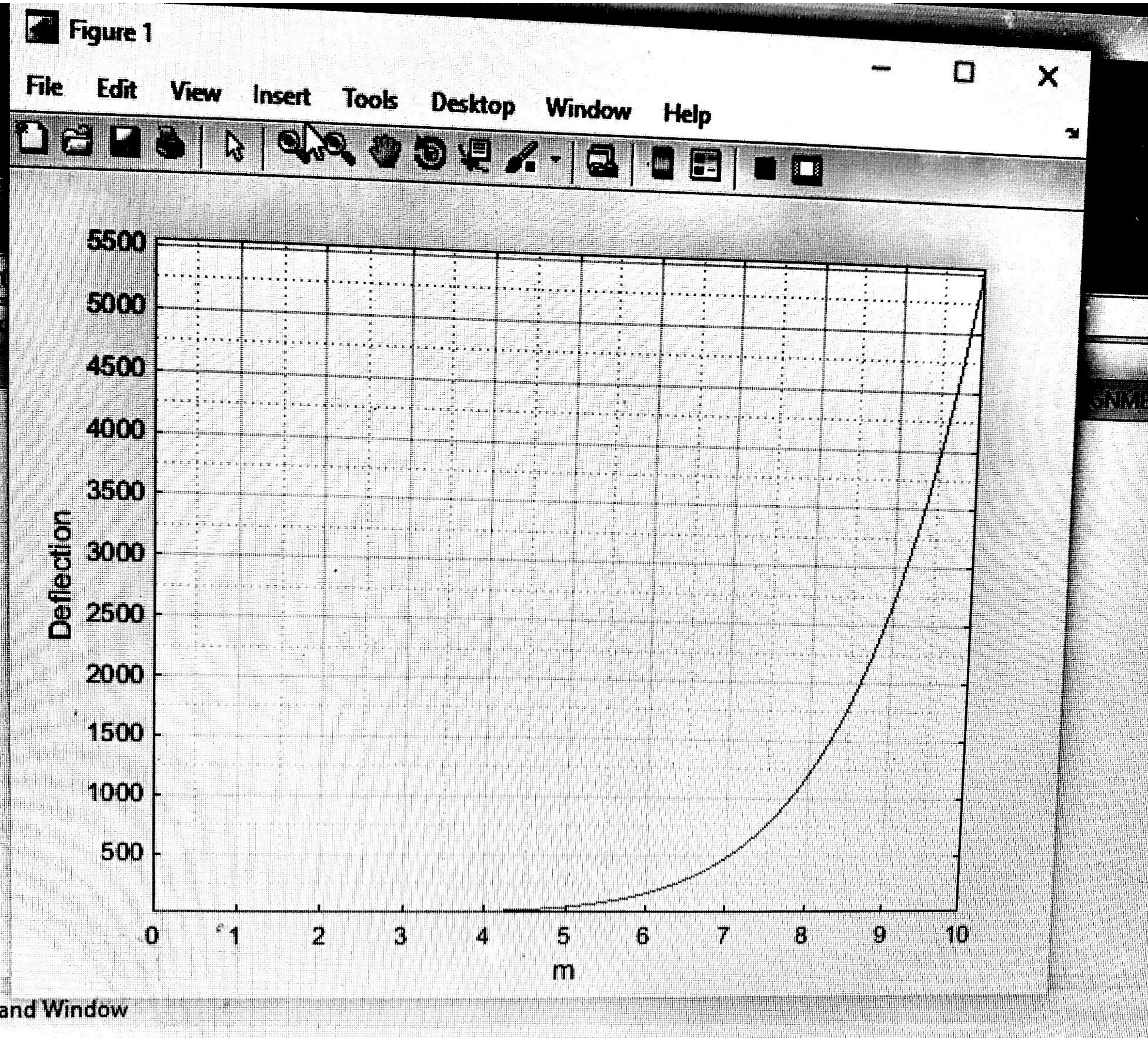
Figure 1
File Edit View Insert Tools Desktop Window Help
5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
Deflection
0
1
2
3
4
5
6
7
8
9
10
m
and Window