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Civil Engineering

17/ENG02/017

ENG 301, Engineering - Maths II

$$1) \quad x(x-1)y'' + (3x-1)y' + y = 0$$

$$x^2y'' - xy'' + 3xy' - y' + y = 0$$

$$y' = w_1' + w_2' + w_3' + w_4' + w_5'$$

$$w_1' = x^2y''$$

$$V = x^2, \quad V^{(1)} = 2x, \quad V^{(2)} = 2, \quad V^{(3)} = 0$$

$$u = y'', \quad u^{(1)} = y^{(3)}, \quad u^{(2)} = y^{(4)}, \quad u^{(3)} = y^{(5)}, \quad u^{(4)} = y^{(6)}$$

$$u^{(1)}V^{(1)} + n u^{(n-1)}V^{(n)} + \frac{n(n-1)}{2!} u^{(n-2)}V^{(2)} + \frac{n(n-1)(n-2)}{3!} u^{(n-3)}V^{(3)}$$

$$= y^{(3)} \cdot 2x + n y^{(n-1)} \cdot 2x + \frac{n(n-1)}{2} y^{(n-2)} \cdot 2 + 0$$

$$y^{n+1} + y^n$$

$$ny^n = y^{n+1} + y^n$$

$$w_1' = x^2y^{n+2} + 2xny^{n+1} + n(n-1)y^n$$

$$w_2' = -xy''$$

$$V = -x, \quad V^{(1)} = -1, \quad V^{(2)} = 0$$

$$u = y'', \quad u^{(1)} = y^{(4)}, \quad u^{(2)} = y^{(6)}, \quad u^{(3)} = y^{(8)}, \quad u^{(4)} = y^{(10)}$$

$$u^{(1)}V^{(1)} + n u^{(n-1)}V^{(n)} + \frac{n(n-1)}{2!} u^{(n-2)}V^{(2)}$$

$$y^{(4)} \cdot -1 + n y^{(n-1)} \cdot -1 + 0$$

$$= -xy^{n+2} - ny^{n+1}$$

$$w_3' = 3xy'$$

$$V = 3x, \quad V^{(1)} = 3, \quad V^{(2)} = 0$$

$$u = y', \quad u^{(1)} = y^{(3)}, \quad u^{(2)} = y^{(5)}, \quad u^{(3)} = y^{(7)}, \quad u^{(4)} = y^{(9)}$$

$$u^{(1)}V^{(1)} + n u^{(n-1)}V^{(n)} + \frac{n(n-1)}{2!} u^{(n-2)}V^{(2)}$$

$$= y^{(3)} \cdot 3x + n y^{(n-1)} \cdot 3 + 0$$

$$= 3xy^{n+1} + 3ny^n$$

$$w_4' = -y'$$

$$u = y', \quad u^{(1)} = y^{(3)}, \quad u^{(2)} = y^{(5)}, \quad u^{(3)} = y^{(7)}, \quad u^{(4)} = y^{(9)}$$

$$V = -1, \quad V^{(1)} = 0$$

$$u^n v^{(n)} + n u^{n-1} v^{(n)} \\ y^{n+1} - 1 + 0 \\ = -y^{n+1}$$

$$w_5^n = y$$

$$v = 1, v^{(n)} = 0$$

$$u = y, u^{(n)} = y^{(n)} = n! y^n, u^n = y^n$$

$$u^n v^{(n)} + n u^{n-1} v^{(n)}$$

$$y^n \cdot 1 + 0$$

$$= y^n$$

$$g^n = w_1^n + w_2^n + w_3^n + w_4^n + w_5^n$$

$$= x^2 y^{n+2} + 2xy y^{n+1} + n(n-1)y^n - xy^{n+2} - ny^{n+1} + 3xy y^{n+1} + 3ny^n - y^{n+1} + y^n$$

① initial condition, $x=0$

$$= 0^2 y^{n+2} + 2(0)y y^{n+1} + n(n-1)y^n - 0y^{n+2} - ny^{n+1} + 3(0)y y^{n+1} + 3ny^n - y^{n+1} + y^n$$

$$= n(n-1)y^n - ny^{n+1} + 3ny^n - y^{n+1} + y^n$$

$$= n(n-1)y^n + y^n + 3ny^n - ny^{n+1} - y^{n+1}$$

$$= y^{n+1}(-n-1) + y^n[n(n-1) + 1 + 3n]$$

$$y^{n+1} = -y^n \frac{[n(n-1) + 1 + 3n]}{(-n-1)}$$

$$y^{n+1} = -y^n \frac{[n^2 - n + 1 + 3n]}{-(n+1)}$$

$$y^{n+1} = -y^n \frac{[n^2 + 2n + 1]}{-(n+1)}$$

$$y^{n+1} = -y^n \frac{[(n+1)^2]}{-(n+1)}$$

$$y^{n+1} = y^n (n+1)$$

$$② y_0^{(n)} = 0.0005, y_0' = 0.0005$$

$$① n=0, y^{(0)} = y^{(0)}$$

$$② n=1, y^{(1)} = 2y^{(0)} = 2y^{(0)} = 0.001$$

$$③ n=2, y^{(2)} = 3y^{(1)} = 6y^{(0)} = 0.003$$

$$④ n=3, y^{(3)} = 4y^{(2)} = 24y^{(0)} = 0.012$$

$$⑤ n=4, y^{(4)} = 5y^{(3)} = 120y^{(0)} = 0.06$$

$$⑥ n=5, y^{(5)} = 6y^{(4)} = 720y^{(0)} = 0.36$$

$$⑦ n=6, y^{(6)} = 7y^{(5)} = 5.040y^{(0)} = 2.52$$

$$⑧ n=7, y^{(7)} = 8y^{(6)} = 40.320y^{(0)} = 20.16$$

Ex. 11
Cub. Equat.
17/10/2018
Page 3/21

$$1) x(x-1)y''$$

$$x^2 y''$$

$$y = w_1^n$$

$$w_1^n = x^2$$

$$v = x$$

$$u = y$$

$$u^n v^{(n)}$$

$$= y^{n+2} x$$

$$w_2^n = x^2 y$$

$$w_3^n = -x y$$

$$v = -x$$

$$u = y^n$$

$$u^n v^{(n)}$$

$$y^{n+2}$$

$$w_4^n = 3x$$

$$v = 3x$$

$$u = y'$$

$$u^n v^{(n)}$$

$$= y^{n+2}$$

$$w_5^n = -y$$

$$u =$$

$$v =$$

$$y = y_0 + x y_0' + \frac{x^2}{2!} y_0'' + \frac{x^3}{3!} y_0''' + \frac{x^4}{4!} y_0^{(4)} + \frac{x^5}{5!} y_0^{(5)} + \frac{x^6}{6!} y_0^{(6)} + \frac{x^7}{7!} y_0^{(7)}$$

$$y = 0.0005 + x \cdot 0.0005 + \frac{x^2}{2} \cdot 0.001 + \frac{x^3}{6} \cdot 0.007 + \frac{x^4}{24} \cdot 0.012 + \frac{x^5}{120} \cdot 0.06 + \frac{x^6}{720} \cdot 0.36 + \frac{x^7}{5040} \cdot 2.52$$

$$+ \frac{x^8}{30240} \cdot 20.16$$

① @ x = 5

$$y = 0.0005 + 5 \cdot 0.0005 + \frac{5^2}{2} \cdot 0.001 + \frac{5^3}{6} \cdot 0.007 + \frac{5^4}{24} \cdot 0.012 + \frac{5^5}{120} \cdot 0.06 + \frac{5^6}{720} \cdot 0.36$$

$$+ \frac{5^7}{5040} \cdot 2.52 + \frac{5^8}{30240} \cdot 20.16$$

$$= \cancel{2.44 \times 10^{-2}} \cdot 0.0005 \times 5^{26} = 7.29 \times 10^{-2} \text{ m}$$

② x = 8

$$y = 0.0005 \left[1 + \frac{8^2}{2} + \frac{8^3 \cdot 6}{6} + \frac{8^4 \cdot 24}{24} + \frac{8^5 \cdot 120}{120} + \frac{8^6 \cdot 720}{720} + \frac{8^7 \cdot 5040}{5040} + \frac{8^8 \cdot 30240}{30240} \right]$$

$$= 0.0005 \times \cancel{2.44 \times 10^{26}} \cdot 8^{26}$$

$$= \cancel{2.44 \times 10^{26}} \cdot 1.62 \times 10^{29} \text{ m}$$

③ x = 10

$$y = 0.0005 \left[10^0 + \frac{10^2}{2} + \frac{10^3 \cdot 6}{6} + \frac{10^4 \cdot 24}{24} + \frac{10^5 \cdot 120}{120} + \frac{10^6 \cdot 720}{720} + \frac{10^7 \cdot 5040}{5040} + \frac{10^8 \cdot 30240}{30240} \right]$$

$$y = 0.0005 \times 10^{26}$$

$$= 5 \times 10^{22}$$

④ comandanada

clear

clc

close all

syms x

x = (0:1:10)

Y = 0.0005 * (1 + x + (x.^2) + (x.^3) + (x.^4) + (x.^5) + (x.^6) + (x.^7) + (x.^8))

Yn = subs(Y)

plot(x, Yn)

xlabel('distance (m)')

ylabel('dynamic response (m)')

grid on

grid minor

