# ENG 391 ASSIGNMENT

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1 $x(x-1)y'' + (3x-1)y' + y = 0$

Taking $x(x-1)y'' = a_1$

$(3x-1)y' = a_2$

$y = a_3$

Considering $a_1$

$U = y''$ $\quad V = x(x-1)$

$U^n = y^{(n+2)}$ $\quad V' = 2x-1$

$U^{n-1} = y^{(n+1)}$ $\quad V'' = 2$

$U^{n-2} = y^n$ $\quad V''' = 0$

Considering $a_2$

$U = y'$ $\quad V = (3x-1)$

$U^n = y^{(n+1)}$ $\quad V' = 3$

$U^{n-1} = y^n$ $\quad V'' = 0$

Considering $a_3$

$U = y$ $\quad V = 1$

$U^n = y^n$ $\quad V' = 0$

$$y^n = U^n V + nV^{(n-1)} \cdot V' + \frac{n(n-1)}{2!} U^{(n-2)} V'' + \frac{n(n-1)(n-2)}{3!} \cdot U^{(n-3)} V''' + 1$$

$a_1^n = y^{(n+2)} \cdot (x^2 - x) + n y^{(n+1)} \times (2x-1) + \frac{n(n-1)}{2!} y^n \cdot 2 + 0$

$a_2^n = y^{(n+1)} \times (3x-1) + n y^n \cdot 3 + 0$

$a_3^n = y^n \times 1 + 0$

Summation of $a_1$, $a_2$ and $a_3$

$y^{(n+2)} \cdot (x^2-x) + n y^{(n+1)} \times (2x-1) + \frac{n(n-1)}{2} y^n \cdot 2 +$

$y^{(n+1)} \times (3x-1) + n y^n \times 3 + y^n = 0$

$(x^2-x) y^{(n+2)} + (2xn-n) y^{(n+1)} + (3x-1) y^{(n+1)} + (n^2-n) y^n + 3n y^n + y^n = 0$

$(x^2-x) y^{(n+1)} + (2xn+3x-n-1) y^{(n+1)} + (n^2-n+3n+1)\ ] y^n = 0$

$(x-1) x y^{(n+2)} + (2xn+3x-n-1) y^{(n+1)} + (n^2+2n+1) y^n = 0$

When $x=0$

$(0-1) 0\, y^{(n+2)} + [2(0)n+3(0)-n-1] y^{(n+1)} + (n^2+2n+1) y^n = 0$

$(-n-1) y^{(n+1)} + (n^2+2n+1) y^n = 0$

$-(n+1)(y^{(n+1)})_0 + (n^2+2n+1)(y^n)_0 = 0$

$[y^{(n+1)}]_0 = \frac{(n^2+2n+1)}{(n+1)} [y^n]_0$

$[y^{(n+1)}]_0 = \frac{(n+1)(n+1)}{n+1} [y^n]_0$

$[y^{(n+1)}]_0 = (n+1)(y^n)_0$

When $n=0$

$[y^{(1)}]_0 = 1 (y^{(0)})_0$

When $n=1$

$(y^{(1+1)})_0 = (1+1)[y^{(1)}]_0$

$(y^{(2)})_0 = 2 [y^{(1)}]_0$

When $n=2$

$(y^{(3)}) = 3 (y^{(2)})_0$

$(y^{(3)})_0 = 6 [y^{(2)}]_0$

When $n=3$

$(y^{(4)})_0 = 4 [y^{(3)}]_0$

$[y^{(4)}]_0 = 24 [y^{(0)}]_0$

When $n=4$

$[y^{(4)}]_0 = (4+1)[y^{(4)}]_0$

$[y^{(4)}]_0 = 120\,[y^{(4)}]_0$

When $n=5$

$[y^{(5)}]_0 = (5+1)[y^{(5)}]_0$

$[y^{(5)}]_0 = 720\,[y^{(5)}]_0$

When $n=6$

$[y^{(6)}]_0 = (6+1)[y^{(6)}]_0$

$[y^{(6)}]_0 = 5040\,[y^{(6)}]_0$

$$y = [y^{(0)}]_0 + x[y^{(1)}]_0 + \frac{x^2}{2!}[y^{(2)}]_0 + \frac{x^3}{3!}[y^{(3)}]_0 + \frac{x^4}{4!}[y^{(4)}]_0 + \frac{x^5}{5!}[y^{(5)}]_0 + \frac{x^6}{6!}[y^{(6)}]_0 + \frac{x^7}{7!}[y^{(7)}]_0$$

$$y = [y^{(0)}]_0 + x[y^{(1)}]_0 + \frac{x^2}{2!}\times 2\,[y^{(2)}]_0 + \frac{x^3}{3!}\times 6\,[y^{(3)}]_0 + \frac{x^4}{4!}\times 24\,[y^{(4)}]_0 + \frac{x^5}{5!}\times 120\,[y^{(5)}]_0 + \frac{x^6}{6!}\times 720\,[y^{(6)}]_0 + \frac{x^7}{7!}\times 5040\,[y^{(7)}]_0$$

$$y = (1+x)[y^{(0)}]_0 + (x^2+x^3+x^4+x^5+x^6+x^7)[y^{(1)}]_0$$

$y(0) = 0.0005\,m, \quad y'(0) = 0.0005$

$$y = (1+x)(0.0005\,m) + (x^2+x^3+x^4+x^5+x^6+x^7)(0.0005)$$

b) When $x = 5\,m, 8\,m$ and $10\,m$

$$y = (1+5)(0.0005) + (5^2+5^3+5^4+5^5+5^6+5^7)(0.0005)$$

$$y = 3\times10^{-3}\,m + 9765.0\,m\,(0.0005)$$

$$y = 3\times10^{-3}\,m + 48.825\,m$$

$$y = 48.828\,m$$

When x = 8m

$Y = 1198(0.0005) + [8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7](0.0005)$

$Y = 4.5 \times 10^{-3} + 1198$

$Y = 1198.0045m$

When x = 10m

$Y = (1+0)(0.0005) + (10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)(0.0005)$

$Y = 5.5 \times 10^{-3} + 1111110.0(0.0005)$

$Y = 5555.5555m$

$Y = 5556m$

2 Command window

```
clear
clc
close all
syms x
x = ((1+x)^(0.0005)) + ((x^2 + x^3 + x^4 + x^5 + x^6 + x^7)^
((0.0005))
t = 0:0.01:10
xt = subs(x, t)
xtn = double(xt)
plot(t, xtn)
x label('t')
y label('x')
grid on
grid minor
axis tight
```