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ENG 381

17/ENG04/023

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Assignment III

Question!

The model for the deformation (y) of a structural element is represented by the expression given in Equation (1):

$$x(x-1)y'' + (3x-1)y' + y = 0 \quad \text{—— (1)}$$

Given that $y(0) = 0.0005m$ and $y'(0) = 0.0005$, applying Leibnitz-Maclaurin method,

(a) Obtain the power series solution of the model up to and including the term in x^7,

(b) Estimate the ~~appropr~~ approximate deformation when $x = 5, 8,$ and $10m$

(c) With the aid of MATLAB mpile program, plot the response of the structural element for $0 \le x \le 10m$

Solution

$$x(x-1)y'' + (3x-1)y' + y = 0$$

a $\quad w_1 = x(x-1)y''$

$$w_2 = (3x-1)y'$$

$$w_3 = y$$

According to Leibnitz's theorem

$$y = U^{(n)}V^{(0)} + nU^{(n-1)}V^{(1)} + \frac{n(n-1)}{2!}U^{(n-2)}V^{(2)} + \frac{n(n-1)(n-2)}{3!}U^{(n-3)}V^{(3)} +$$

$$\frac{n(n-1)(n-2)(n-3)}{4!}U^{(n-4)}V^{(4)} + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!}U^{(n-5)}V^{(5)}$$

w_1

$u = y^{(2)}$ $\qquad\qquad$ $v = x^2 - x$

$u^{n} = y^{(n+2)}$ $\qquad\qquad$ $v^{(1)} = 2x - 1$

$u^{(n-1)} = y^{(n+1)}$ $\qquad\qquad$ $v^{(2)} = 2$

$u^{(n-2)} = y^{(n)}$ $\qquad\qquad$ $v^{(3)} = 0$

w_2

$u = y^{(1)}$ $\qquad\qquad$ $v = (3x - 1)$

$u^{(n)} = y^{(n+1)}$ $\qquad\qquad$ $v^{(1)} = 3$

$u^{(n-1)} = y^{(n)}$ $\qquad\qquad$ $v^{(2)} = 0$

$w_3 \quad = y^{(n)}$

$$0 = (x^2 - x)y^{(n+2)} + n(2x-1)y^{(n+1)} + \frac{n(n-1)}{2!}2y^{(n)} + (3x-1)y^{(n+1)}$$
$$+ n3y^{(n)} + y^{(n)}$$

$$= x^2y^{(n+2)} - xy^{(n+2)} + 2xny^{(n+1)} - ny^{(n+1)} + n(n-1)y^{(n)} + 3xy^{(n+1)} - y^{(n+1)}$$
$$+ 3ny^{(n)} + y^{(n)}$$

Assume $x = 0$

$$0 = -n(y^{(n+1)})_0 + n(n-1)(y^{(n)})_0 - (y^{(n+1)})_0 + 3n(y^{(n)})_0 + (y^{(n)})_0$$

$$(-n-1)(y^{(n+1)})_0 + (n^2+2n+1)(y^{(n)})_0 = 0$$

$$(n^2+2n+1)(y^{(n)})_0 = -(-n-1)(y^{(n+1)})_0$$

$$y^{(n+1)} = \frac{(n^2-2n+1)}{-(-n-1)}(y^{(n)})_0$$

given that $(y^{(0)})_0 = 0.0005m$ and $(y^{(1)})_0 = 0.0005$.

to find $y^{(2)}$, $n = 1$

$$(y^{(2)})_0 = \frac{(1^2 + 2(1) + 1)}{-(-1-1)}(y^{(1)})_0 = \frac{4}{2}(y^{(1)})_0 = 2y^{(1)} = 0.001$$

When $n=2$

$$\left(y^{(3)}\right)_0 = \frac{(2^2+2(2)+1)}{-(-2-1)}\left(y^{(2)}\right)_0 = \frac{9}{3}\left(y^{(2)}\right)_0 = 3\left(y^{(2)}\right)_0$$

$$= 3\times 0.001 = 0.003$$

When $n=3$

$$\left(y^{(4)}\right)_0 = \frac{(3^2+2(3)+1)}{-(-3-1)}\left(y^{(3)}\right)_0 = \frac{16}{4}\left(y^{(3)}\right)_0 = 4\left(y^{(3)}\right)_0$$

$$= 4\times 0.003 = 0.012$$

When $n=4$

$$\left(y^{(5)}\right)_0 = \frac{(4^2+2(4)+1)}{-(-4-1)}\left(y^{(4)}\right)_0$$

$$= \frac{25}{5}\left(y^{(4)}\right)_0 = 5\times 0.12 = 0.06$$

When $n=5$

$$\left(y^{(6)}\right)_0 = \frac{(5^2+2(5)+1)}{-(-5-1)}\left(y^{(5)}\right)_0 = \frac{36}{6}\left(y^{(5)}\right)_0 = 6\left(y^{(5)}\right)_0$$

$$= 6\times 0.06 = 0.36$$

$$\left(y^{(7)}\right)_0 = \frac{(6^2+2(6)+1)}{-(-6-1)}\left(y^{(6)}\right)_0 = \frac{49}{7}\left(y^{(6)}\right)_0 = 7\times 0.36 = 2.52$$

Maclaurin's theorem

$$y = \left(y^{(0)}\right)_0 + x\left(y^{(1)}\right)_0 + \frac{x^2}{2!}\left(y^{(2)}\right)_0 + \frac{x^3}{3}\left(y^{(3)}\right)_0 + \frac{x^4}{4!}\left(y^{(4)}\right)_0 + \frac{x^5}{5!}\left(y^{(5)}\right)_0 + \frac{x^6}{6!}\left(y^{(6)}\right)_0 + \frac{x^7}{7!}\left(y^{(7)}\right) + \cdots$$

$$y = 0.0005 + 0.0005x + \frac{0.0001x^2}{2!} + \frac{0.0003x^3}{3!} + \frac{0.012x^4}{4!}$$
$$+ \frac{0.06x^5}{5!} + \frac{0.36x^6}{6!} + \frac{2.52x^7}{7!}$$

b When $x = 5$

$$y(5) = 0.005 + 0.0005(5) + 0.0005(5)^2 + 0.0005(5)^3 + 0.0005(5)^4 + 0.0005(5)^5 + 0.0005(5)^6 + 0.0005(5)^7$$
$$= 48m$$

When $x = 8$

$$y(8) = 0.0005 + 0.0005(8) + 0.0005(8)^2 + 0.0005(8)^3 + 0.0005(8)^4 + 0.0005(8)^5 + 0.0005(8)^6 + 0.0005(8)^7$$

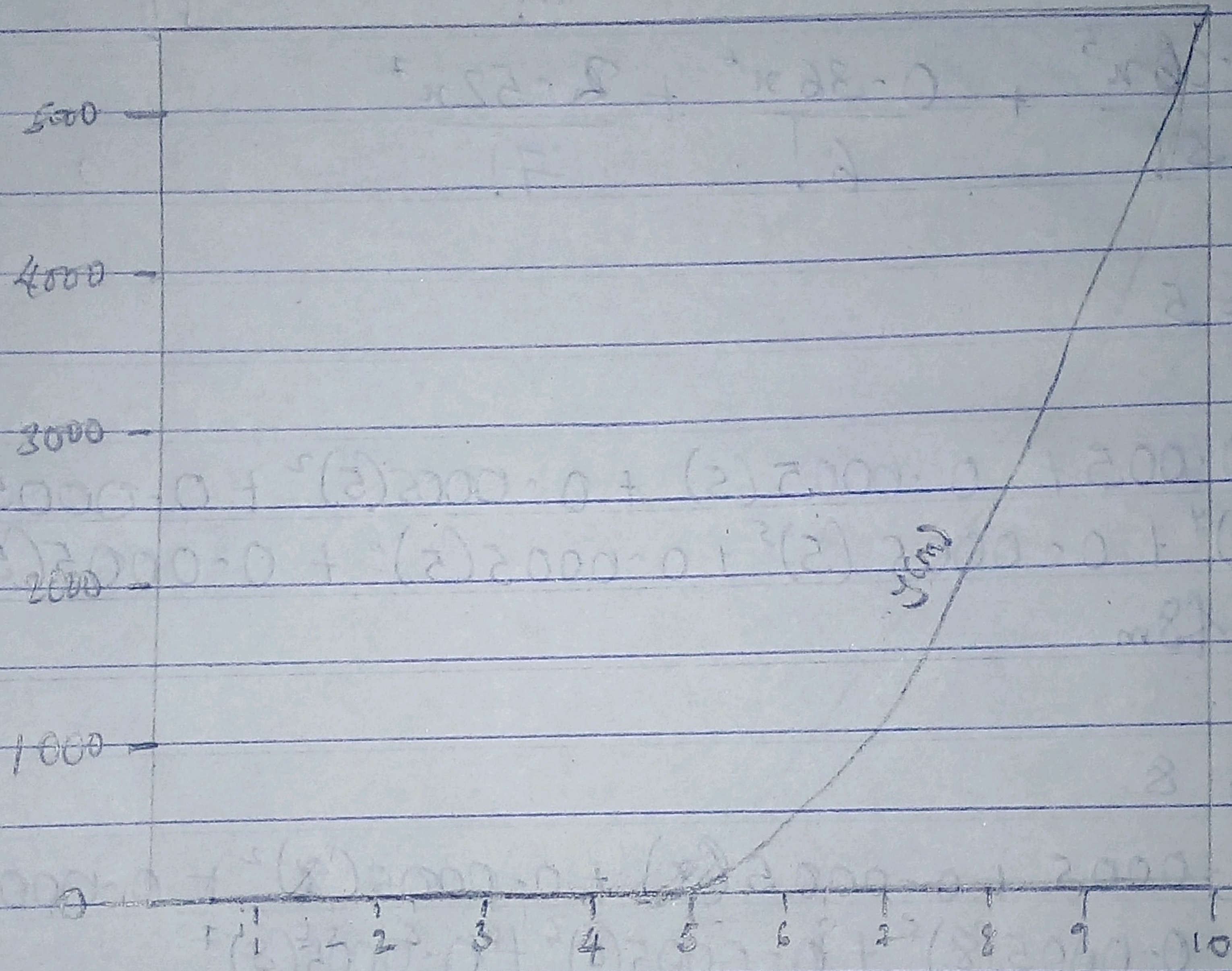
$$= 1198.37m$$

When $x = 10$

$$y(10) = 0.0005 + 0.0005(10) + 0.0005(10)^2 + 0.0005(10)^3 + 0.0005(10)^4 + 0.0005(10)^5 + 0.0005(10)^6 + 0.0005(10)^7$$

$$= 5555.55m$$

c
1. Commandwindow
2. Clear
3. Clc
4. Close all
5. y = 0.0005*(1+x+x^2+x^3+x^5+x^6+x^7)
6. x = 0:1:10
7. plot (x, y)
8. legend ('y(m)')
9. Grid minor
10. Grid on


5000
4000
3000
2000
1000
0
1
2
3
4
5
6
7
8
9
10
y(m)