

NAME: SULE MUBARAK
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QUESTION I

Find the area bounded
and the ordinates at

Solution

$$A = \int_1^3 y_2 \cdot dx - \int_1^3 y_1 \cdot dx$$

$$y_2 = 3e^{2x} \quad y_1 = 3e^{-x}$$

$$A = \int_1^3 3e^{2x} dx - \int_1^3 3e^{-x} dx$$

$$= 3 \int_1^3 e^{2x} dx - 3 \int_1^3 e^{-x} dx$$

$$= 3 \left[\frac{e^{2x}}{2} \right]_1^3 - 3 \left[\frac{e^{-x}}{-1} \right]_1^3$$

$$= 3 \left[\frac{e^{2(3)}}{2} - \frac{e^{2(1)}}{2} \right] -$$

$$= 3 \left[\frac{e^4}{2} - \frac{e^2}{2} \right] - 3 \left[\frac{e^{-3}}{2} - \frac{e^{-1}}{2} \right]$$

$$= 3(23.605) - 3$$

$$= 70.12 \text{ square}$$

QUESTION II

The parametric equation

$$x = 2 + 2t - 2\cos^2 \frac{t}{10} \text{ t. F}$$

between $t=0$ and $t=2\pi$

Solution

$$y = 2 \sin \frac{\pi}{10} t \quad x = 2 + 2t$$

$$A = \int_a^b y \cdot dx$$

$$dx/dt = 2 + 2 \frac{\pi}{10} \sin \frac{\pi}{10} t$$

$$A = \int_0^{10} 2 \sin \frac{\pi}{10} t \cdot \left(2 + \frac{2\pi}{10} \sin \frac{\pi}{10} t \right) dt$$

$$= \int_0^{10} 4 \sin \frac{\pi}{10} t + \frac{2\pi}{5} \sin^2 \frac{\pi}{10} t dt$$

$$= 4 \int_0^{10} \sin \frac{\pi}{10} t + \frac{2\pi}{5} \int_0^{10} \sin^2 \frac{\pi}{10} t dt$$

Recall: $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$$\therefore \sin^2 \frac{\pi}{10} = \frac{1}{2} \left(1 - \cos \frac{2\pi}{10} \right)$$

$$A = 4 \int_0^{10} \sin \frac{\pi}{10} t + \frac{2\pi}{5} \int_0^{10} \left(\frac{1}{2} (1 - \cos \frac{2\pi}{10} t) \right) dt$$

$$= 4 \left[-\frac{10}{\pi} \cos \frac{\pi}{10} t \right]_0^{10} + \frac{2\pi}{5} \left[\frac{t}{2} - \frac{10}{2\pi} \sin \frac{2\pi}{10} t \right]_0^{10}$$

$$= 4 \left[\frac{-10}{\pi} \cos \frac{\pi}{10} (10) - \left(-\frac{10}{\pi} \cos \frac{\pi}{10} (0) \right) \right] + \frac{2\pi}{5} \left[\frac{10}{2} - \frac{10}{2\pi} \sin \frac{2\pi}{10} (10) - \left(\frac{0}{2} - \frac{10}{2\pi} \sin \frac{2\pi}{10} (0) \right) \right]$$

$$= 4 \left[\frac{-10}{\pi} \cos \pi + \frac{10}{\pi} \right] + \frac{2\pi}{5} \left[\frac{10}{2} - \frac{10}{2\pi} \sin 2\pi - \left(\frac{0}{2} - \frac{10}{2\pi} \sin 0 \right) \right]$$

$$= \frac{-40}{\pi} \cos \pi + \frac{40}{\pi}$$

$$= \frac{40}{\pi} + \frac{40}{\pi}$$

$$= \frac{80}{\pi}$$

$$= 31.75 \text{ square}$$