BRIGGS FRANCIS S01B1
17/ENG04/015

a) $x(x-1)y'' + (3x-1)y' + y = 0$

Expanding the bracket

$(x^2-x)y'' + (3x-1)y' + y = 0$

$\omega_1 = (x^2-x)y''$; $\omega_2 = (3x-1)y'$; $\omega_3 = y$

using Leibnitz for ω_1, ω_2 and ω_3

$^nC_0 u^{(n)}v^{(0)} + {^nC_1} u^{(n-1)}v^{(1)} + {^nC_2} u^{(n-2)}v^{(2)} + \ldots$

ω_1

$u = y^{(2)}$ $\quad V = x^2 - x$

$u^{(n)} = y^{(2+n)}$ $\quad v^{(1)} = 2x - 1$

$u^{(n-1)} = y^{(1+n)}$ $\quad v^{(2)} = 2$

$u^{(n-2)} = y^{(n)}$ $\quad v^{(3)} = 0$

$\omega_1 = y^{(n+2)}.(x^2-x) + n.y^{(n+1)}.(2x-1) + \frac{n(n-1)}{2!}y^{(n)}.2 + 0 \ldots$

ω_2

$u = y^{(1)}$ $\quad V = 3x - 1$

$u^{(n)} = y^{(1+n)}$ $\quad v^{(1)} = 3$

$u^{(n-1)} = y^{(n)}$ $\quad v^{(2)} = 0$

$\omega_2 = y^{(n+1)}.(3x-1) + 3ny^{(n)}$

ω_3

$u = y$ $\quad V = 1$

$u^{(n)} = y^{(n)}$ $\quad v^{(1)} = 0$

$\omega_3 = y^n$

$\omega_1 + \omega_2 + \omega_3 = 0$

$y^{(n+2)}.(x^2-x) + (2nx-n)y^{(n+1)} + y^{(n)}(n^2-n) + y^{(n+1)}.(3x-1) + 3ny^{(n)} + y^{(n)} = 0$

assume $x = 0$

$\Rightarrow (-n)y^{(n+1)} + y^{(n)}(n^2-n) + 3ny^{(n)} + y^{(n)} = 0$

collecting like terms

$y^{(n+1)}(-n-1) + y^{(n)}(n^2-n+3n+1) = 0$

$y^{(n+1)}(-n-1) + y^{(n)}(n^2+2n+1) = 0$

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$-y^{(n+1)}(n+1) + y^{(n)}(n^2+2n+1) = 0$

$y^{(n+1)}(n+1) = y^{(n)}(n^2+2n+1)$

$y^{(n+1)}\cancel{(n+1)} = y^{(n)}(\cancel{(n+1)}(n+1))$

$(y^{(n+1)})_0 = (y^{(n)})_0(n+1)$ [recurrence relation]

assume $n = 0$

$(y^{(1)})_0 = (y^{(0)})_0(1)$

$0.005 = 0.005(1)$

$n = 1$

$(y^{(2)})_0 = 2(y^{(1)})_0$

$n = 2$

$(y^{(3)})_0 = 3(y^{(2)})_0 \Rightarrow 3(2(y^{(1)})_0) = 6(y^{(1)})_0$

$n = 3$

$(y^{(4)})_0 = 4(y^{(3)})_0 \Rightarrow 4(3(2(y^{(1)})_0)) = 24(y^{(1)})_0$

$n = 4$

$(y^{(5)})_0 = 5(y^{(4)})_0 \Rightarrow 5(4(3(2(y^{(1)})_0))) = 120(y^{(1)})_0$

Macllaurin series

$y = (y^{(0)})_0 + (y^{(1)})_0 x + \frac{(y^{(2)})_0 x^2}{2!} + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^{(4)})_0$

$+ \frac{x^5}{5!}(y^{(5)})_0 + \frac{x^6}{6!} \cancel{y}\, 6!\,(y^{(1)})_0 + \frac{x^7}{7!}\, 7!\,(y^{(7)})_0$

$y = 0.0005 + 0.0005x + \frac{\cancel{2}x^2(y^{(1)})_0}{\cancel{2!}} + \frac{\cancel{6}x^3}{\cancel{3!}}(y^{(1)})_0 + \frac{\cancel{24}x^4}{\cancel{4!}}(y^{(1)})_0$

$\frac{\cancel{120}x^5}{\cancel{5!}}(y^{(1)})_0 + \dots$

$y = 0.0005(1+x) + (x^2+x^3+x^4+x^5 + x^6 + x^7)\,0.0005$

$(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)\,0.0005 =$

b) when $x = 5m$

$y = [1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7]\,0.0005$

$y = \cancel{1.953}m$ $48.82\dot{3}\,m$

when $x = 8m$

$y = [1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7]\,0.0005$

$y = \cancel{18.72m}$ $1198.37\dot{2}\,m$

when $x = 10m$

$y = [1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7]\,0.0005$

$y = 5555.556m$