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17/ENG05/038
Mechatronics
ENG 381

1. $x(x-1)y'' + (3x-1)y' + y = 0$

Expanding the bracket

$(x^2-x)y'' + (3x-1)y' + y = 0$

$kl_1 = (x^2-x)y''$

$kl_2 = (3x-1)y'$

$kl_3 = y$

Using Leibnitz theorem

Fo $U^nV + nU^{n-1}V^{-1} + \frac{n(n-1)}{2!}U^{n-2}V^2 + \frac{n(n-1)(n-2)}{3!}U^{n-3}V^3 + \dots$

For kl_1	For kl_2	For kl_3
$u = y^2 \quad v = x^2 - x$	$U = y' \quad V = 3x-1$	$U = y \quad V = 1$
$u^n = y^{n+2} \quad v' = 2x-1$	$U^n = y^{n+1} \quad V' = 3$	$U^n = y^n \quad V' = 0$
$u^{n-1} = y^{n+1} \quad v^2 = 2$	$y^{n-1} = y^n \quad V^2 = 0$	
$u^{n-2} = y^n \quad v^3 = 0$		

$kl_1 = y^{(n+2)}\cdot(x^2-x) + n\cdot y^{(n+1)}(2x-1) + \frac{n(n-1)}{2}y^n\cdot 2$

$= (x^2-x)y^{(n+2)} + n(2x-1)y^{(n+1)} + n(n-1)y^n$

$kl_2 = y^{(n+1)}\cdot(3x-1) + 3ny^n$

$kl_3 = y^n$

Summing all together

$(x^2-x)y^{(n+2)} + n(2x-1)y^{(n+1)} + (n^2-n)y^n + y^{(n+1)}\cdot(3x-1) + 3ny^n + y^n = 0$

assuming $x = 0$

$(0-0)y^{(n+2)} + (2n(0)-n)y^{(n+1)} + (n^2-n)y^n + (3(0)-1)y^{(n+1)} + 3ny^n + y^n = 0$

$= -ny^{(n+1)} + (n^2-n)y^n - y^{(n+1)} + 3ny^n + y^n = 0$

Collecting like terms

$= y^{(n+1)}(-n-1) + y^n(n^2-n+3n+1) = 0$

$= -y^{n+1}(n+1) + y^n(n^2+2n+1) = 0$

$(n+1)y^{(n+1)} = (n^2+2n+1)y^n$ $\qquad [n^2+2n+1 = (n+1)(n+1)]$

$\therefore (n+1)y^{(n+1)} = (n+1)(n+1)y^n$

Divide both sides by $(n+1)$

$y^{(n+1)} = (n+1)y^n$ - Recurrence relation

$(y^{n+1})_0 = y^n(n+1)$

$(y^0)_0 = 0.0005$

$(y^1)_0 = 0.0005$

When $n = 0$

$[y^{(0+1)}]_0 = (0+1)(y^0)_0$

$[y^{(1)}]_0 = 1[y^0]_0$

$n=1;\ [y^{(1+1)}]_0 = (1+1)(y^1)_0$

$[y^{(2)}]_0 = 2(y^1)_0$

$n=2;\ (y^3)_0 = (2+1)y^{(2)}$

$= 3[y^2]_0 = 3[2(y^{(1)}]_0 = 6(y^1)_0$

$n=3;\ (y^4)_0 = (3+1)y^3$

$= 4[y^3]_0 = 4[6(y^1)_0] = 24(y^1)_0$

$n=4;\ (y^5)_0 = (4+1)y^4$

$= 5(y^4)_0 = 5[24(y^1)_0] = 120(y^1)_0$

$n=5;\ (y^6)_0 = (5+1)y^5$

$= 6(y^5)_0 = 6[120(y^1)_0] = 720(y^1)_0$

$n=6;\ (y^7)_0 = (6+1)y^6$

$= (y^7) = 7(y^6) = 7[720(y^1)_0] = 5040(y^1)_0$

Using macClarin series.

$$Y = (Y^0)_0 + x(Y')_0 + \frac{x^2}{2!}(Y^2)_0 + \frac{x^3}{3!}(Y^3)_0 + \frac{x^4}{4!}(Y^4)_0 + \frac{x^5}{5!}(Y^5)_0 + \frac{x^6}{6!}(Y^6)_0 + \frac{x^7}{7!}(Y^7)_0 + \ldots$$

$$Y = (Y^0)_0 + x(Y^0) + \frac{x^2}{2!}(2Y')_0 + \frac{x^3}{3!}(6Y')_0 + \frac{x^4}{4!}(24y')_0 + \frac{x^5}{5!}(120y') + \frac{x^6}{6!}(720y')_0 + \frac{x^7}{7!}(5040Y')$$

$$Y = Y^0(1+x) + (x^2+x^3+x^4+x^5+x^6+x^7)Y'$$

$$Y = 0.0005(1+x) + (x^2+x^3+x^4+x^5+x^6+x^7)0.0005$$

ii. Estimate the approximate deformation when $x = 5, 8$ and $10m$:

When $x = 5m$

$$Y = Y^0(1+5) + (5^2+5^3+5^4+5^5+5^6+5^7)0.0005$$

$$= 0.0005(6) + (25+125+625+3125+15625+78125)0.0005$$

$$Y = 48.828m$$

When $x = 8m$

$$Y = Y^0(1+8) + (8^2+8^3+8^4+8^5+8^6+8^7)Y'$$

$$Y = 0.0005(1+8) + (64+512+4096+32768+262144+2097152)0.000$$

$$= 1198.3725m$$

When $x = 10m$

$$Y = Y^0(1+10) + (10^2+10^3+10^4+10^5+10^6+10^7)Y'$$

$$= 0.0005(11) + (100+1000+10000+100\,000+1000\,000+10\,000\,000)0.00$$

$$= 5555.56m$$

MatLab mFile

Command Window

Clear

CLC

Close all

x = 0:0.01:10

Y = (0.0005*(1+x)) + ((x.^2 + x.^3 + x.^4 + x.^5 + x.^6 + x.^7)*0.0005)

Yn = subs(Y)

plot (x, Yn)

x Label ('m')

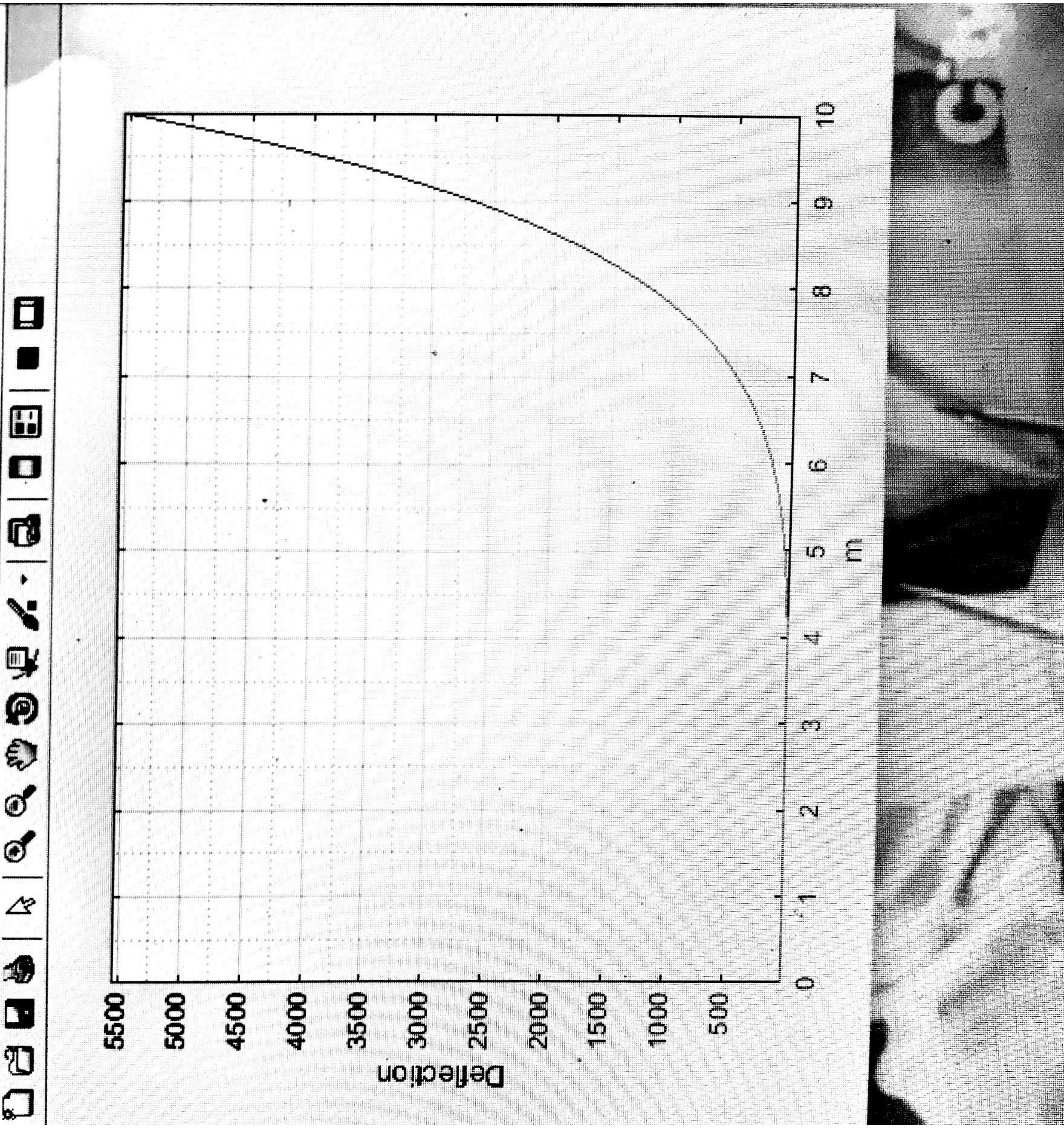
Y Label ('Deflection')

axis tight

grid on

grid minor

Scanned with CamScanner


Deflection
5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
1
2
3
4
5
6
7
8
9
10
m