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Question 1

Find the Area bounded by the curves $y = e^{2x}$ and $y = e^{-x}$ and the ordinates at $x=1$ and $x=2$.

Solution

$$A = \int_1^2 y_2 dx - \int_1^2 y_1 dx$$

$$y_2 = 3e^{2x} \quad y_1 = 3e^{-x}$$
$$A = \int_1^2 3e^{2x} dx - \int_1^2 3e^{-x} dx$$

$$= 3 \int_1^2 e^{2x} dx - 3 \int_1^2 e^{-x} dx$$

$$= 3 \left[\frac{e^{2x}}{2} \right]_1^2 - 3 \left[\frac{e^{-x}}{-1} \right]_1^2 + c$$

$$= 3 \left[\frac{e^{2x}}{2} \right]_1^2 - 3 \left[\frac{e^{-x}}{-1} \right]_1^2 + c$$

$$= 3 \left[\frac{e^{2(2)}}{2} - \frac{e^{2(1)}}{2} \right] - 3 \left[\frac{e^{-2}}{-1} - \frac{e^{-1}}{-1} \right]$$

$$= 3 \left[\frac{e^4}{2} - \frac{e^2}{2} \right] - 3 \left[-e^{-2} + e^{-1} \right]$$

$$= 3(23.605) - 3(0.2325)$$

$$= 70.12 \text{ square units}$$

Question 2

The Parametric equations of a curve $y = 2 \sin \pi/10 t$ and $x = 2 + 2t = 2 \cos \pi/10 t$. Find the area under the curve between $t=0$ and

Solution

$$y = 2 \sin \pi t \quad x = 2 + 2t = 2 \cos \pi/10 t$$

$$A = \int_{t_2}^{t_1} y dx$$

$$d\text{oc}|dt = 2 + 2\pi/10 \sin^2 \pi/10 t$$

$$A = \int_0^{10} 2 \sin \pi/10 t \cdot (2 + \frac{2\pi}{10} \sin^2 \pi/10 t) dt$$

$$= \int_0^{10} 4 \sin \pi t + \frac{2\pi}{5} \sin^2 \pi t dt$$

$$= 4 \int_0^{10} \sin \pi t + \frac{2\pi}{5} \int_0^{10} \sin^2 \pi t dt$$

Recall: $\sin^2 a = \frac{1}{2} (1 - \cos 2a)$
 $\therefore \sin^2 \pi/10 t = \frac{1}{2} (1 - \cos \pi/5 t)$

$$A = 4 \int_0^{10} \sin \pi t + \frac{2\pi}{5} \int_0^{10} \left(\frac{1}{2} (1 - \cos \pi/5 t) \right) dt$$

$$= 4 \left[\frac{10}{\pi} \cos \frac{\pi}{10} t \right]_0^{10} + \frac{2\pi}{10} \left[t - \frac{5}{\pi} \sin \frac{\pi}{5} t \right]_0^{10}$$

$$= 4 \left[\frac{-10}{\pi} \cos \pi (10) - \left(\frac{-10}{\pi} \cos \pi (0) \right) \right] + \frac{2\pi}{10} \left[10 - 5 \sin 2\pi - \left(\frac{10}{\pi} - \frac{5}{\pi} \sin \pi \right) \right]$$

$$\left(\frac{10 - 5 \sin \pi}{\pi} \right)$$

$$= 4 \left[\frac{-10 \cos \pi + 10}{\pi} \right] + \frac{4\pi}{10} \left[10 - \frac{5 \sin 2\pi}{\pi} \right]$$

$$= \frac{-40 \cos \pi + 40}{\pi} + \frac{4\pi}{10} (10 - \sin 2\pi)$$

$$= \frac{40}{\pi} + \frac{40}{\pi} + 2\pi$$

$$= \frac{80}{\pi} + 2\pi$$

≈ 31.75 square units.