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Assignment Number 3: Engineering Mathematics.

\* The model for the deformation ( $y$ ) of a structural element is represented by the expression given in Equation (1)

$$x(x-1)y'' + (3x-1)y' + y = 0.$$

Given that  $y(0) = 0.0025m$  and  $y'(0) = 0.0005$ , applying Leibnitz Maclaurin's Method.

- (a) Obtain the power series solution of the model up to and including the term in  $x^7$
- (b) Estimate the approximate deformation when  $x = 5, 8$  and  $10m$  and.
- (c) Write the end of a MATLAB m-file program, plot the response of the structural element for  $0 \leq x \leq 10m$ .

Solution:

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Given that  $y(0) = 0.0025$ , applying Leibnitz, Maclaurin method, obtain power series up to  $x^7$ .

$$xy''(x-1) + (3x-1)y' + y = 0$$

$$xy^{(n+2)}(x-1) + y^{(n+1)}(3x-1) + y^{(n)} = 0$$

$$xy^{(n+2)}(x-1) + y^{(n+1)}(3x-1) + y^{(n)} = 0$$

a<sub>1</sub>

$$u = y''$$

$$v = x(x-1) = x^2 - x$$

$$u^n = y^{(n+2)}$$

$$v' = 2x - 1$$

$$u^{(n-1)} = y^{(n+1)}$$

$$v'' = 2$$

$$u^{(n-2)} = y^{(n)}$$

$$v''' = 0$$

a<sub>2</sub>

$$u = y'$$

$$v = 3x - 1$$

$$u^n = y^{(n+1)}$$

$$v' = 3$$

$$u^{(n-1)} = y^{(n)}$$

$$v'' = 0$$

a<sub>3</sub>

$$u = y^{(0)}$$

$$v = 1$$

$$u^n = y^{(n)}$$



Applying Leibnitz.

$$y^{(n)} = y^{(n)} + n \frac{y^{(n-1)}}{1} + \frac{n(n-1)}{2!} y^{(n-2)} + \frac{n(n-1)(n-2)}{3!} y^{(n-3)} + \frac{n(n-1)(n-2)(n-3)}{4!} y^{(n-4)} + \dots$$

$$a_1 = y^{(n+2)}(x^2-x) + ny^{(n+1)}(2x-1) + \frac{n(n-1)}{2!} y^{(n)}(2) + 0$$

$$a_2 = y^{(n+1)}(3x-1) + ny^{(n)}(3) + 0$$

$$a_3 = y^{(n)} - 1 + 0$$

$$y^{(n)} = y^{(n+2)}(x^2-x) + ny^{(n+1)}(2x-1) + \frac{n(n-1)}{2!} y^{(n)} + y^{(n+1)}(3x-1) + ny^{(n)}(3) + y^{(n)} = 0$$

$$y^{(n)} = y^{(n+2)}(x^2-x) + ny^{(n+1)}(2x-1) + \frac{n(n-1)}{2!} y^{(n)} + y^{(n+1)}(3x-1) + 3ny^{(n)} + y^{(n)} = 0$$

$$y^{(n)} = y^{(n+2)}(x^2-x) + (2xn-n)y^{(n+1)} + (3x-1)y^{(n+1)} + (n^2-n)y^{(n)} + 3ny^{(n)} + y^{(n)} = 0$$

$$y^{(n)} = y^{(n+2)}(x^2-x) + (2xn+3x-n-1)y^{(n+1)} + (n^2-n+3n+1)y^{(n)} = 0$$

$$y^{(n)} = y^{(n+2)}(x^2-x) + (2xn+3x-n-1)y^{(n+1)} + (n^2+2n+1)y^{(n)} = 0$$

$$(x-1)x y^{(n+2)} + (2xn+3x-n-1)y^{(n+1)} + (n^2+2n+1)y^{(n)} = 0$$

when  $x=0$

$$(0-1) \cdot 0 \cdot y^{(n+2)} + (2(0)n+3(0)-n-1)y^{(n+1)} + (n^2+2n+1)y^{(n)} = 0$$

$$(-n-1)y^{(n+1)} + (n^2+2n+1)y^{(n)} = 0$$

$$-(n+1) \left[ y^{(n+1)} \right]_0 + (n^2+2n+1) \left[ y^{(n)} \right]_0 = 0$$

$$-(n+1) \left[ y^{(n+1)} \right]_0 = -(n^2+2n+1) \left[ y^{(n)} \right]_0$$

divide through by  $-(n+1)$

$$\frac{-(n+1) \left[ y^{(n+1)} \right]_0}{-(n+1)} = \frac{-(n^2+2n+1)}{-(n+1)} \left[ y^{(n)} \right]_0$$

$$\left[ y^{(n+1)} \right]_0 = \frac{(n^2+2n+1)}{(n+1)} \left[ y^{(n)} \right]_0$$

when  $n=0$

$$\left[ y^{(0+1)} \right]_0 = \frac{(0^2+2(0)+1)}{(0+1)} \left[ y^{(0)} \right]_0$$



$$[y^{(1)}]_0 = \frac{1}{1} [y^{(0)}]_0$$

$$[y^{(1)}]_0 = 1 [y^{(0)}]_0$$

• when  $n=1$ .

$$[y^{(1+1)}]_0 = \frac{(1^2 + 2(1) + 1)}{(1+1)} [y^{(1)}]_0$$

$$[y^{(2)}]_0 = \frac{1+2+1}{1+1} [y^{(1)}]_0$$

$$[y^{(2)}]_0 = \frac{4}{2} [y^{(1)}]_0 \\ = 2 [y^{(1)}]_0 //$$

• when  $n=2$

$$[y^{(2+1)}]_0 = \frac{(2^2 + 2(2) + 1)}{(2+1)} [y^{(2)}]_0$$

$$[y^{(3)}]_0 = \frac{9}{3} [y^{(2)}]_0$$

$$[y^{(3)}]_0 = 3 [y^{(2)}]_0 = (3)(2) [y^{(1)}]_0 = 6 [y^{(1)}]_0 //$$

• when  $n=3$

$$[y^{(3+1)}]_0 = \frac{(3^2 + 2(3) + 1)}{(3+1)} [y^{(3)}]_0$$

$$[y^{(4)}]_0 = \frac{16}{4} [y^{(3)}]_0$$

$$[y^{(4)}]_0 = 4 [y^{(3)}]_0 = (4)(6) [y^{(1)}]_0 = 24 [y^{(1)}]_0 //$$

• when  $n=4$

$$[y^{(4+1)}]_0 = \frac{(4^2 + 2(4) + 1)}{(4+1)} [y^{(4)}]_0$$

$$[y^{(5)}]_0 = \frac{25}{5} [y^{(4)}]_0 = 5 [y^{(4)}]_0$$

$$= (5)(24) [y^{(1)}]_0 \\ = 120 [y^{(1)}]_0 //$$



when  $n=5$ .

$$[y^{(5+1)}]_0 = \frac{(5^2 + 2(5) + 1)}{(5+1)} [y^{(5)}]_0$$

$$[y^{(6)}]_0 = \frac{36}{6} [y^{(5)}]_0 = 6 [y^{(5)}]_0 = (6)(120) [y^{(1)}]_0 = 720 [y^{(1)}]_0$$

when  $n=6$

$$[y^{(6+1)}]_0 = \frac{(6^2 + 2(6) + 1)}{(6+1)} [y^{(6)}]_0$$

$$[y^{(7)}]_0 = \frac{49}{7} [y^{(6)}]_0 = 7 [y^{(6)}]_0 = (7)(720) [y^{(1)}]_0 = 5040 [y^{(1)}]_0$$

⇒ Applying Maclaurin's theorem:

$$y = [y^{(0)}]_0 + x [y^{(1)}]_0 + \frac{x^2}{2!} [y^{(2)}]_0 + \frac{x^3}{3!} [y^{(3)}]_0 + \frac{x^4}{4!} [y^{(4)}]_0 + \frac{x^5}{5!} [y^{(5)}]_0 + \frac{x^6}{6!} [y^{(6)}]_0 + \frac{x^7}{7!} [y^{(7)}]_0$$

$$y = [y^{(0)}]_0 + x [y^{(1)}]_0 + \frac{x^2}{2!} [2(y^{(1)})]_0 + \frac{x^3}{3!} [6(y^{(1)})]_0 + \frac{x^4}{4!} [24(y^{(1)})]_0 + \frac{x^5}{5!} [120(y^{(1)})]_0 + \frac{x^6}{6!} [720(y^{(1)})]_0 + \frac{x^7}{7!} [5040(y^{(1)})]_0$$

$$y = [y^{(0)}]_0 (1+x) + x^2 [y^{(1)}]_0 + x^3 [y^{(1)}]_0 + x^4 [y^{(1)}]_0 + x^5 [y^{(1)}]_0 + x^6 [y^{(1)}]_0 + x^7 [y^{(1)}]_0$$

Factorizing:

$$y = (1+x) [y^{(0)}]_0 + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7) [y^{(1)}]_0$$

1)  $y(0) = 0.0005m, y'(0) = 0.0005$

$$y = (1+x)(0.0005m) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)(0.0005)$$

\* when  $x=5m, 8m \& 10m$ .

$$y = (1+5m)(0.0005m) + (5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)(0.0005)$$

$$y = 3 \times 10^{-3} + 97650m(0.0005)$$

$$y = 3 \times 10^{-3} + 48.825$$

$$y = 48.828m$$

\* when  $x=8m$

$$y = (1+8m)(0.0005m) + (8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)(0.0005)$$

$$y = 4.5 \times 10^{-3} + 2396736(0.0005)$$

$$y = 4.5 \times 10^{-3} + 1198.368$$

$$y = 1198.3725$$

$$y = 1198m$$



\* when  $x=10m$

$$y = (1+10m)(0.0005) + (10^2+10^3+10^4+10^5+10^6+10^7)(0.0005)$$

$$y = 5.5 \times 10^{-3} + 11111100(0.0005)$$

$$y = 5.5 \times 10^{-3} + 5555.55$$

$$y = 5555.5555$$

$$y \approx 5556m //$$

\* Matlab

Command window

clear

clc

close all

Syms x

$$x = ((1+x) * (0.0005)) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7) * (0.0005)$$

$$t = 0 : 0.01 : 10$$

$$xt = \text{subs}(x, t)$$

$$xtn = \text{double}(xt)$$

Plot(t, xtn)

xlabel('t')

ylabel('x')

grid on

grid minor

axis tight

PLS TURN OVER

