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 Course - ECE 6571  
 Assignment II

ii) Solution

$$x(x-1)y'' + (3x-1)y' + y = 0$$

for the nth derivative

$$x(x-1)y^{(n+2)} + (3x-1)y^{(n+1)} + y^{(n)} = 0$$

$$3y^{(n+1)} = 0$$

$$x(x-1)y^{(n+2)} + (3x-1)y^{(n+1)} + y^{(n)} = 0$$

$$\text{at } x=0$$

$$(y^{(n+2)})_0 + (y^{(n+1)})_0 + (y^{(n)})_0 = 0$$

$$(y^{(n+2)})_0 + (y^{(n+1)})_0 = -y^{(n)}_0$$

$$(y^{(n+1)})_0 + (y^{(n)})_0 = -y^{(n-1)}_0$$

$$(y^{(n)})_0 + (y^{(n-1)})_0 = -y^{(n-2)}_0$$

$$(y^{(n-1)})_0 + (y^{(n-2)})_0 = -y^{(n-3)}_0$$

$$(y^{(n-2)})_0 + (y^{(n-3)})_0 = -y^{(n-4)}_0$$

$$(y^{(n-3)})_0 + (y^{(n-4)})_0 = -y^{(n-5)}_0$$

$$(y^{(n-4)})_0 + (y^{(n-5)})_0 = -y^{(n-6)}_0$$

$$(y^{(n-5)})_0 + (y^{(n-6)})_0 = -y^{(n-7)}_0$$

$$(y^{(n-6)})_0 + (y^{(n-7)})_0 = -y^{(n-8)}_0$$

$$(y^{(n-7)})_0 + (y^{(n-8)})_0 = -y^{(n-9)}_0$$

$$(y^{(n-8)})_0 + (y^{(n-9)})_0 = -y^{(n-10)}_0$$

$$(y^{(n-9)})_0 + (y^{(n-10)})_0 = -y^{(n-11)}_0$$

$$(y^{(n-10)})_0 + (y^{(n-11)})_0 = -y^{(n-12)}_0$$

$$(y^{(n-11)})_0 + (y^{(n-12)})_0 = -y^{(n-13)}_0$$

$$(y^{(n-12)})_0 + (y^{(n-13)})_0 = -y^{(n-14)}_0$$

$$(y^{(n-13)})_0 + (y^{(n-14)})_0 = -y^{(n-15)}_0$$

$$(y^{(n-14)})_0 + (y^{(n-15)})_0 = -y^{(n-16)}_0$$

$$(y^{(n-15)})_0 + (y^{(n-16)})_0 = -y^{(n-17)}_0$$

$$(y^{(n-16)})_0 + (y^{(n-17)})_0 = -y^{(n-18)}_0$$

$$(y^{(n-17)})_0 + (y^{(n-18)})_0 = -y^{(n-19)}_0$$

$$(y^{(n-18)})_0 + (y^{(n-19)})_0 = -y^{(n-20)}_0$$

$$(y^{(n-19)})_0 + (y^{(n-20)})_0 = -y^{(n-21)}_0$$

$$(y^{(n-20)})_0 + (y^{(n-21)})_0 = -y^{(n-22)}_0$$

$$(y^{(n-21)})_0 + (y^{(n-22)})_0 = -y^{(n-23)}_0$$

$$(y^{(n-22)})_0 + (y^{(n-23)})_0 = -y^{(n-24)}_0$$

$$(y^{(n-23)})_0 + (y^{(n-24)})_0 = -y^{(n-25)}_0$$

$$(y^{(n-24)})_0 + (y^{(n-25)})_0 = -y^{(n-26)}_0$$

$$(y^{(n-25)})_0 + (y^{(n-26)})_0 = -y^{(n-27)}_0$$

at  $n=1$

$$(y^{(2)})_0 = -1(y^{(1)})_0$$

$$(y^{(1)})_0 = 2(y^{(0)})_0$$

$$(y^{(0)})_0 = 2(y^{(0)})_0$$

$$(y^{(1)})_0 = (2+1)(y^{(0)})_0$$

$$(y^{(2)})_0 = 3(y^{(0)})_0$$

$$(y^{(3)})_0 = 6(y^{(0)})_0$$

$$\text{at } n=2$$

$$(y^{(4)})_0 = (3+1)(y^{(3)})_0$$

$$(y^{(3)})_0 = 4(y^{(2)})_0 = 4 \times 6(y^{(0)})_0 = 24(y^{(0)})_0$$

$$\text{at } n=3$$

$$(y^{(5)})_0 = (4+1)(y^{(4)})_0$$

$$5 \times 24(y^{(0)})_0$$

$$(y^{(6)})_0 = 120(y^{(0)})_0$$

$$\text{at } n=4$$

$$(y^{(7)})_0 = (5+1)(y^{(6)})_0$$

$$6 \times 120(y^{(0)})_0$$

$$(y^{(8)})_0 = 720(y^{(0)})_0$$

$$\text{at } n=5$$

$$(y^{(9)})_0 = (6+1)(y^{(8)})_0$$

$$8 \times 720(y^{(0)})_0$$

$$(y^{(10)})_0 = 5760(y^{(0)})_0$$



$$C_j \cdot 2 = (5+1)C_j \cdot 0$$

$$= 2C_j \cdot 0$$

$$C_j \cdot 0 = 720C_j \cdot 0$$

$$n = 6$$

$$C_j \cdot 0 = (6+1)C_j \cdot 0$$

$$= 7C_j \cdot 0$$

$$= 7 + 720C_j \cdot 0$$

$$C_j \cdot 0 = 504C_j \cdot 0$$

from mechanism

$$y = C_j \cdot 0 + 2C_j \cdot 0 + \frac{x^2}{2!} C_j \cdot 0 + \frac{20^2}{2!} C_j \cdot 0 + \frac{x^4}{4!} C_j \cdot 0 + \dots$$

$$\therefore y = C_j \cdot 0 + 2C_j \cdot 0 + \frac{x^2}{2!} (2C_j \cdot 0) + \frac{20^2}{2!} (2C_j \cdot 0) + \frac{x^4}{4!} (2C_j \cdot 0) + \frac{x^2}{2!} (720C_j \cdot 0) + \frac{20^2}{2!} (720C_j \cdot 0) + \dots$$

$$y = C_j \cdot 0 + C_j \cdot 0 [2x + 20^2 + 20^4 + 20^6 + 20^8 + 20^{10} + x^2 + x^4 + \dots]$$

$$\text{but } C_j \cdot 0 = 0.0005 \text{ m and } C_j \cdot 0 = 0.0005$$

$$\therefore y = 0.0005 + 0.0005 [2x + 20^2 + 20^4 + 20^6 + 20^8 + 20^{10} + x^2 + x^4 + \dots]$$

here  $20 = 5m$

$$y = 0.0005 + 0.0005 [5 + 25 + 125 + 625 + 3125 + 15625 + 78125]$$

$$y = 0.0005 + 0.0005 [97655]$$

$$\therefore y = 48.828m$$

where  $20 = 8m$

$$y = 0.0005 + 0.0005 [8 + 4 + 512 + 4096 + 32768 + 262144 + 2097]$$

$$y = 1178.3728m$$

when  $x = 10m$