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17/ENG04/043 ~~Elect~~ Elet Engineering
Assignment III

The model for the deformation (y) of a structural element is represented by the expression given in equation (1):

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Given that $y(0) = 0.0005m$ and $y'(0) = 0.0005$, applying Leibnitz-Maclaurin method.

(a) Obtain the power series solution of the model up to and including the term in x^7

(b) Estimate the approximate deformation when $x = 5$, 8 and 10m and

(c) With the aid of a MATLAB mfile program, plot the response of the structural element for $0 \leq x \leq 10m$.

Solution.

(a) $(x^2-x)y'' + (3x-1)y' + y = 0$

$w_1 = (x^2-x)y''$ $\quad$ $w_2 = (3x-1)y'$ $\quad$ $w_3 = y$

$u = y^{(2)}$ $\quad v = x^2 - x$ $\quad$ $u = y^{(1)}$ $\quad v = 3x-1$ $\quad$ $u = y$ $\quad v = 1$

$u^n = y^{(n+2)}$ $\quad v' = 2x-1$ $\quad$ $u^n = y^{(n+1)}$ $\quad v' = 3$ $\quad$ $u^n = y^n$ $\quad v' = 0$

$u^{n-1} = y^{(n+1)}$ $\quad v'' = 2$ $\quad$ $u^{n-1} = y^{(n)}$ $\quad v'' = 0$

$u^{n-2} = y^{(n)}$ $\quad v''' = 0$

using Leibnitz theorem

$$u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v'' + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v''' + \dots$$

$w_1 + w_2 + w_3 = 0$

$$y^{(n+2)}(x^2-x) + n y^{(n+1)}(2x-1) + \frac{n(n-1)}{\cancel{2!}} y^n (\cancel{2}) + y^{(n+1)}(3x-1) + n y^n \cdot 3 + y^n = 0$$

assuming $x = 0$

$$n(y^{(n+1)})_0(-1) + n(n-1)(y^n)_0 + (y^{(n+1)})_0(-1) + 3n(y^n)_0 + (y^n)_0 = 0$$

$$-n(y^{n+1})_0 - (y^{n+1})_0 + n^2 - n(y^n)_0 + 3n(y^n)_0 + (y^n)_0 = 0$$

$$(y^{n+1})_0(-n-1) + (y^n)_0(n^2 - n + 3n + 1) = 0$$

$$(-n-1)(y^{n+1})_0 = -(n^2+2n+1)(y^n)_0 = 0$$

$$(y^{n+1})_0 = \frac{\cancel{t}(n^2+2n+1)}{\cancel{t}(n+1)}(y^n)_0$$

$$(y^{n+1})_0 = \frac{n^2+2n+1}{n+1}(y^n)_0$$

$$(y^0)_0 = 0.0005m$$

$$(y^1)_0 = 0.0005m$$

when $n = 1$

$$y^{1+1} = (y^{(2)})_0 = \frac{1^2+2(1)+1}{1+1}(y^{(1)})_0 = \frac{4}{2}(y^{(1)})_0 = 2(y^{(1)})_0 = 2\times 0.0005$$
$$= 0.001$$

when $n = 2$

$$y^{2+1} = (y^{(3)})_0 = \frac{2^2+2(2)+1}{2+1}(y^{(2)})_0 = 3(y^{(2)})_0 = 3\times 0.001$$
$$= 0.003$$

when $n=3$

$$y^{3+1} = (y^{(4)})_0 = \frac{3^2+2(3)+1}{3+1}(y^{(3)})_0 = 4(y^{(3)})_0 = 4\times 0.003$$
$$= 0.012$$

when $n=4$

$$y^{4+1} = (y^{(5)})_0 = \frac{4^2+2(4)+1}{4+1}(y^{(4)})_0 = 5(y^{(4)})_0 = 5\times 0.012$$
$$= 0.06$$

when $n = 5$

$$y^{5+1} = (y^{(6)})_0 = \frac{5^2+2(5)+1}{5+1}(y^{5})_0 = 6(y^{(5)})_0 = 6\times 0.06$$
$$= 0.36$$

when $n=6$

$$y^{6+1} = (y^{(7)})_0 = \frac{6^2+2(6)+1}{6+1}(y^{(6)})_0 = 7(y^{(6)})_0 = 7\times 0.36$$
$$= 2.52$$

maclaurin series ;

$$y = (y^0)_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^{(4)})_0 + \frac{x^5}{5!}(y^{(5)})_0 + \frac{x^6}{6!}(y^{(6)})_0 + \frac{x^7}{7!}(y^{(7)})_0 + \cdots$$

$$y = 0.0005 + 0.0005x + \frac{x^2}{2!}.0.001 + \frac{x^3}{3!}.0.003 + \frac{x^4}{4!}.0.012 + \frac{x^5}{5!}.0.06 + \frac{x^6}{6!}.0.36 + \frac{x^7}{7!}.2.52 + \cdots$$

$$y = 0.0005 + 0.0005x + 0.0005x^2 + 0.0005x^3 + 0.0005x^4 + 0.0005x^5 + 0.0005x^6 + 0.0005x^7 + \cdots$$

$y = 0.0005(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \dots)$

b) when $x = 5m$

$y = 0.0005(1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)$

$y = 48.828m$

when $x = 8m$

$y = 0.0005(1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$

$y = 1198.3725m$

when $x = 10m$

$y = 0.0005(1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$

$y = 5555.5555m$

(c) Commandwindow

```
clear
clc
close all
syms x
y = 0.0005 *(1+x+x^2+x^3+x^4+x^5+x^6+x^7)
x = 0:1:10
yn = subs(y)
ynn = double(yn)
plot(x, ynn)
grid on
grid minor
```