* The model for the deformation (y) of a structural element is represented by the expression given in Equation (1)

$$x(x-1)y'' + (3x-1)y' + y = 0.$$

Given that $y(0) = 0.0005m$ and $y'(0) = 0.0005$, applying leibnitz Maclaurin Method.

(a) Obtain the power series solution of the model up to and including the term in x^7

(b) estimate the approximate deformation when $x = 5, 8$ and $10m$ and.

(c) with the aid of a MATLAB mfile program, plot the response of the structural element for $0 \le x \le 10m$.

Solution.

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Given that $y(0) = 0.0005$, applying leibnitz, maclaurin method, obtain power series up to x^7.

$$xy''(x-1) + (3x-1)y' + y = 0$$

$$xy^{(2)}(x-1) + y^{(1)}(3x-1) + y^{(0)} = 0$$

$$xy^{(n+2)}(x-1) + y^{(n+1)}(3x-1) + y^{(0)} = 0$$

- a_1

$u = y''$	$v = x(x-1) = x^2 - x$
$u^n = y^{(n+2)}$	$v' = 2x-1$
$u^{(n-1)} = y^{(n+1)}$	$v'' = 2$
$u^{(n-2)} = y^{(n)}$	$v''' = 0$

- a_2

$u = y'$	$v = 3x-1$
$u^n = y^{(n+1)}$	$v' = 3$
$u^{(n-1)} = y^n$	$v'' = 0$

- a_3

$u = y^{(0)}$	$v = 1.$
$u^n = y^{(n)}$	

Applying leibnitz.

$$y^{(n)} = u^{(n)}v + n u^{(n-1)}v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3 + \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} v^4 + \dots$$

$$a_1 = y^{(n+2)}(x^2 x) + n y^{(n+1)}(2x-1) + \frac{n(n-1)}{2!} y^{(n)}(2) + 0$$

$$a_2 = y^{(n+1)}(3x-1) + n y^{(n)} 3 + 0$$

$$a_3 = y^{(n)} \cdot 1 + 0$$

$$y^{(n)} = y^{(n+2)} x(x-1) + n y^{(n+1)}(2x-1) + n(n-1) y^{(n)} + y^{(n+1)}(3x-1) + n y^{(n)}(3) + y^{(n)} = 0$$

$$y^{(n)} = y^{(n+2)}(x^2-x) + n y^{(n+1)}(2x-1) + n(n-1) y^{(n)} + y^{(n+1)}(3x-1) + 3n y^{(n)} + y^{(n)} = 0$$

$$y^{(n)} = y^{(n+2)}(x^2-x) + (2xn-n) y^{(n+1)} + (3x-1) y^{(n+1)} + (n^2-n) y^{(n)} + 3n y^{(n)} + y^{(n)} = 0$$

$$y^{(n)} = y^{(n+2)}(x^2-x) + (2xn+3x-n-1) y^{(n+1)} + (n^2-n+3n+1) y^{(n)} = 0$$

$$y^{(n)} = y^{(n+2)}(x^2-x) + (2xn+3x-n-1) y^{(n+1)} + (n^2+2n+1) y^{(n)} = 0$$

$$(x-1)x y^{(n+2)} + (2xn+3x-n-1) y^{(n+1)} + (n^2+2n+1) y^{(n)} = 0$$

when $x = 0$

$$\cancel{(0-1)0}\, y^{(n+2)} + (\cancel{2(0)n+3(0)}-n-1) y^{(n+1)} + (n^2+2n+1) y^{(n)} = 0$$

$$(-n-1) y^{(n+1)} + (n^2+2n+1) y^{(n)} = 0$$

$$-(n+1)\left[y^{(n+1)}\right]_0 + (n^2+2n+1)\left[y^{(n)}\right]_0 = 0$$

$$-(n+1)\left[y^{(n+1)}\right]_0 = -(n^2+2n+1)\left[y^{(n)}\right]_0$$

divide through by $-(n+1)$

$$\frac{\cancel{-(n+1)}\left[y^{(n+1)}\right]_0}{\cancel{-(n+1)}} = \frac{\cancel{-}(n^2+2n+1)}{\cancel{-}(n+1)}\left[y^{(n)}\right]_0$$

$$\left[y^{(n+1)}\right]_0 = \frac{(n^2+2n+1)}{(n+1)}\left[y^{(n)}\right]_0$$

when $n = 0$

$$\left[y^{(0+1)}\right]_0 = \frac{(0^2+2(0)+1)}{(0+1)}\left[y^{(0)}\right]_0$$

$$\left[y^{(1)}\right]_0 = \frac{1}{1}\left[y^{(0)}\right]_0$$

$$\left[y^{(1)}\right]_0 = 1\left[y^{(0)}\right]_0 .$$

- when $n=1$.

$$\left[y^{(1+1)}\right]_0 = \frac{(1^2+2(1)+1)}{(1+1)}\left[y^{(1)}\right]_0$$

$$\left[y^{(2)}\right]_0 = \frac{1+2+1}{1+1}\left[y^{(1)}\right]_0$$

$$\left[y^{(2)}\right]_0 = \frac{\cancel{4}}{\cancel{2}}\left[y^{(1)}\right]_0$$

$$= 2\left[y^{(1)}\right]_0$$

- when $n=2$

$$\left[y^{(2+1)}\right]_0 = \frac{(2^2+2(2)+1)}{(2+1)}\left[y^{(2)}\right]_0$$

$$\left[y^{3}\right]_0 = \frac{9}{3}\left[y^{(2)}\right]_0$$

$$\left[y^{(3)}\right]_0 = 3\left[y^{(2)}\right]_0 = (3)(2)\left[y^{(1)}\right]_0 = 6\left[y^{(1)}\right]_0$$

- When $n=3$

$$\left[y^{(3+1)}\right]_0 = \frac{(3^2+2(3)+1)}{(3+1)}\left[y^{(3)}\right]_0$$

$$\left[y^{(4)}\right]_0 = \frac{\cancel{16}}{\cancel{4}}\left[y^{(3)}\right]_0$$

$$\left[y^{(4)}\right]_0 = 4\left[y^{(3)}\right]_0 = (4)(6)\left[y^{(1)}\right]_0 = 24\left[y^{(1)}\right]_0 .$$

- when $n=4$

$$\left[y^{(4+1)}\right]_0 = \frac{(4^2+2(4)+1)}{(4+1)}\left[y^{(4)}\right]_0$$

$$\left[y^{(5)}\right]_0 = \frac{\cancel{25}^{5}}{\cancel{5}}\left[y^{(4)}\right]_0 = 5\left[y^{(4)}\right]_0$$

$$= (5)(24)\left[y^{(1)}\right]_0$$

$$= 120\left[y^{(1)}\right]_0 .$$

when $n=5$.

$$[y^{(5+1)}]_0 = \frac{(5^2+2(5)+1)}{(5+1)}[y^{(5)}]_0$$

$$[y^{(6)}]_0 = \frac{36}{6}[y^{(5)}]_0 = 6[y^{(5)}]_0 = (6)(120)[y^{(1)}]_0 = \underline{720[y^{(1)}]_0}$$

when $n=6$

$$[y^{(6+1)}]_0 = \frac{(6^2+2(6)+1)}{(6+1)}[y^{(6)}]_0$$

$$[y^{(7)}]_0 = \frac{49}{7}[y^{(6)}]_0 = 7(y^{(6)})_0 = (7)(720)[y^{(1)}]_0 = \underline{5040[y^{(1)}]_0}$$

⇒ applying maclaurin theorem

$$y = [y^{(0)}]_0 + x[y^{(1)}]_0 + \frac{x^2}{2!}[y^{(2)}]_0 + \frac{x^3}{3!}[y^{(3)}]_0 + \frac{x^4}{4!}[y^{(4)}]_0 + \frac{x^5}{5!}[y^{(5)}]_0 + \frac{x^6}{6!}[y^{(6)}] + \frac{x^7}{7!}[y^{(7)}]_0$$

$$y = [y^{(0)}]_0 + x[y^{(0)}]_0 + \frac{x^2}{\cancel{2!}}\cdot[\cancel{2}(y^{(1)})]_0 + \frac{x^3}{\cancel{3!}}\cdot[\cancel{6}[y^{(1)}]]_0 + \frac{x^4}{\cancel{4!}}\cdot[\cancel{24}(y^{(1)})]_0 + \frac{x^5}{\cancel{5!}}\cdot[\cancel{120}(y^{(1)})]_0 + \frac{x^6}{\cancel{6!}}\cdot[\cancel{720}(y^{(1)})]_0 + \frac{x^7}{\cancel{7!}}[\cancel{5040}\, y^{(1)}]_0$$

$$y = [y^{(0)}]_0(1+x) + x^2[y^{(1)}]_0 + x^3[y^{(1)}]_0 + x^4[y^{(1)}]_0 + x^5[y^{(1)}]_0 + x^6[y^{(1)}]_0 + x^7[y^{(1)}]_0.$$

factorizing:

$$y = (1+x)[y^{(0)}]_0 + (x^2+x^3+x^4+x^5+x^6+x^7)$$

) $y(0) = 0.0005m$, $y'(0) = 0.0005$

$$y = (1+x)(0.0005m) + (x^2+x^3+x^4+x^5+x^6+x^7)(0.0005)$$

* when $x = 5m$, $8m$ & $10m$.

$$y = (1+5m)(0.0005m) + (5^2+5^3+5^4+5^5+5^6+5^7)(0.0005)$$

$$y = 3\times10^{-3} + 97650m(0.0005)$$

$$y = 3\times10^{-3} + 48.825$$

$$y = 48.828m$$

* when $x = 8m$

$$y = (1+8m)(0.0005m) + (8^2+8^3+8^4+8^5+8^6+8^7)(0.0005)$$

$$y = 4.5\times10^{-3} + 2396736(0.0005)$$

$$y = 4.5\times10^{-3} + 1198.368$$

$$y = 1198.3725$$

$$y = 1198m$$

* when $x = 10m$

$y = (1+10m)(0.0005) + (10^2+10^3+10^4+10^5+10^6+10^7)(0.0005)$

$y = 5.5\times10^{-3} + 11111100(0.0005)$

$y = 5.5\times10^{-3} + 5555.55$

$y = 5555.5555$

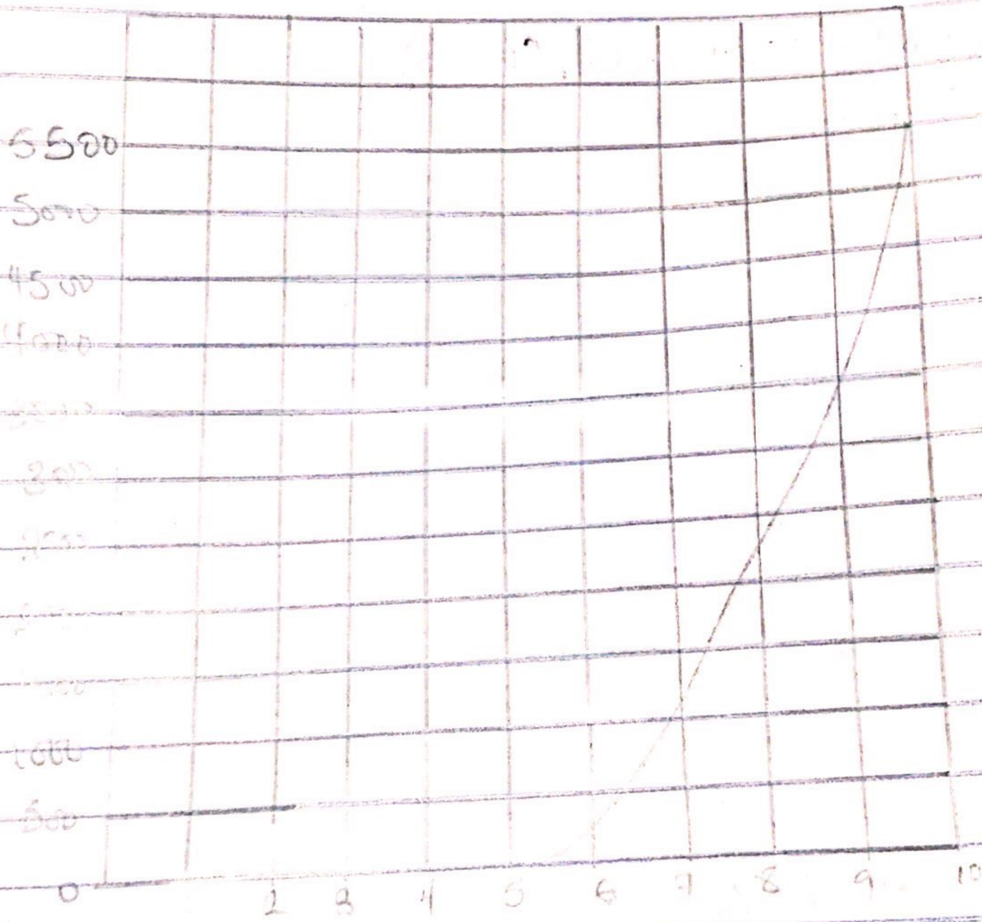
$y \simeq 5556m$ //

*Matlab.

Commandwindow

```
clear
clc
close all
syms x
x=((1+x)*(0.0005)) +((x^2 +x^3+x^4 +x^5+x^6 +x^7)*(0.0005))
t= 0:0.01:10
xt= subs(x,t)
xtn = double(xt)
Plot(t, xtn)
xlabel ('t')
y label ('x')
grid on
grid minor
axis tight
```

PLS TURN OVER


5500
5000
4500
1000
0
2
3
4
5
6
7
8
9
10