

1. Solution

(a)  $x(x-1)y'' + (3x-1)y' + y = 0$

for the  $n$ th derivative,

$$x(x-1)y^{n+2} + (2x-1)ny^{n+1} + n(n-1)y^n + (3x-1)y^{n+1} + 3ny^n + y^n = 0$$

$$x(x-1)y^{n+2} + y^{n+1}(2xn - n + 3x - 1) + y^n(n^2 - n + 3n + 1) = 0$$

at  $x = 0$ ,

$$(y^{n+1})_0(-n-1) + (y^n)_0(n^2 + 2n + 1) = 0$$

$$-(n+1)(y^{n+1})_0 = -(n^2 + 2n + 1)(y^n)_0$$

$$(n+1)(y^{n+1})_0 = (n^2 + 2n + 1)(y^n)_0$$

$$(y^{n+1})_0 = (n+1)(y^n)_0$$

at  $n = 0$ ,

$$(y^{0+1})_0 = (0+1)(y^0)_0$$

$$(y^1)_0 = (y)_0$$

at  $n = 1$ ,

$$(y^{1+1})_0 = (1+1)(y^1)_0$$

$$(y^2)_0 = 2(y^1)_0$$

at  $n = 2$ ,

$$(y^{2+1})_0 = (2+1)(y^2)_0$$

$$(y^3)_0 = 3(y^2)_0$$

$$(y^3)_0 = 6(y^1)_0$$

$$\text{at } n=3,$$
$$(y^3)_0 = (3+1)(y^2)_0$$

$$(y^4)_0 = 4(y^3)_0 = 4 \times 6(y^2)_0 = 24(y^2)_0$$

$$\text{at } n=4,$$
$$(y^5)_0 = (4+1)(y^4)_0$$

$$= 5(y^4)_0$$
$$= 5 \times 24(y^2)_0$$
$$(y^5)_0 = 120(y^2)_0$$

$$\text{at } n=5,$$
$$(y^6)_0 = (5+1)(y^5)_0$$
$$= 6(y^5)_0$$
$$= 6 \times 120(y^2)_0$$
$$(y^6)_0 = 720(y^2)_0$$

$$\text{at } n=6,$$
$$(y^7)_0 = (6+1)(y^6)_0$$
$$= 7(y^6)_0$$
$$= 7 \times 720(y^2)_0$$
$$(y^7)_0 = 5040(y^2)_0$$

from maclaurin series,

$$y = (y)_0 + x(y^1)_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \dots$$

$$\therefore y = (y)_0 + x(y^1)_0 + \frac{x^2}{2!}(2(y^2)_0) + \frac{x^3}{3!}(6(y^2)_0) + \frac{x^4}{4!}(24(y^2)_0)$$

$$+ \frac{x^5}{5!}(120(y^2)_0) + \frac{x^6}{6!}(720(y^2)_0) + \frac{x^7}{7!}(5040(y^2)_0) + \dots$$

$$y = (y)_0 + x(y')_0 + x^2(y'')_0 + x^3(y''')_0 + x^4(y^{(4)})_0 + x^5(y^{(5)})_0 + x^6(y^{(6)})_0 + x^7(y^{(7)})_0 + \dots$$

$$y = (y)_0 + (y')_0 [x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \dots]$$

but  $(y)_0 = 0.0005_m$  and  $(y')_0 = 0.0005,$

$$\therefore y = 0.0005 + 0.0005 [x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7]$$

(b) When  $x = 5_m,$

$$y = 0.0005 + 0.0005 [5 + 25 + 125 + 625 + 3125 + 15625 + 78125]$$

$$y = 0.0005 + 0.0005 [97655]$$

$$\therefore y = 48.828_m$$

When  $x = 8_m,$

$$y = 0.0005 + 0.0005 [8 + 64 + 512 + 4096 + 32768 + 262144 + 2097152]$$

$$y = 0.0005 + 0.0005 [2396744]$$

$$\therefore y = 1198.3725_m$$

When  $x = 10_m,$

$$y = 0.0005 + 0.0005 [10 + 100 + 1000 + 10000 + 100000 + 1000000 + 10000000]$$

$$y = 0.0005 + 0.0005 [11111110]$$

$$y = 5555.5555_m$$

```

(c) command window
clear
clc
syms x
syms y = x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7;
x = (0:10);
y = 0.0005 + 0.0005 * (x + (x.^2) + (x.^3) + (x.^4) + (x.^5) +
(x.^6) + (x.^7));
plot(x, y)
grid on
grid minor
xlabel('x')
ylabel('Structural - Deformation')

```