at $n = 5$,

$$(y^6)_0 = (5+1)(y^5)_0$$
$$= 6(y^5)_0$$
$$= 6 \times 120 (y')_0$$
$$(y^6)_0 = 720 (y')_0$$

at $n = 6$

$$(y^7)_0 = (6+1)(y^6)_0$$
$$= 7(y^6)_0$$
$$= 7 \times 720 (y')_0$$
$$(y^7)_0 = 5040 (y')_0$$

from maclaurin series

$$y = (y)_0 + x(y')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \dots$$

$$\therefore y = (y)_0 + x(y')_0 + \frac{x^2}{2!}[2(y')_0] + \frac{x^3}{3!}(6(y')_0) + \frac{x^4}{4!}(24(y'))$$
$$+ \frac{x^5}{5!}(120(y')_0) + \frac{x^6}{6!}(720(y')_0) + \frac{x^7}{7!}(5040(y')_0) + \dots$$

$$y = (y)_0 + x(y')_0 + x^2(y')_0 + x^3(y')_0 + x^4(y')_0 + x^5(y')_0 + x^6(y')_0 + x^7(y')_0 + \dots$$

$$y = (y)_0 + (y')_0[x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \dots]$$

but $(y) = 0.0005m$ and $(y')_0 = 0.0005$,

$$\therefore y = 0.0005 + 0.0005[x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7]$$

i) when $x = 5m$,

$$y = 0.0005 + 0.0005[5 + 25 + 125 + 625 + 3125 + 15625 + 78125]$$
$$y = 0.0005 + 0.0005[97655]$$
$$\therefore y = 48.828m$$

when $x = 8m$.

$$y = 0.0005 + 0.0005[8 + 64 + 512 + 4096 + 32768 + 262144 + 2097152$$
$$y = 0.0005 + 0.0005[2396744]$$
$$\therefore y = 1198.3725m$$

when x=10m

y) $y = 0.0005 \text{ to } 0.0005 [10 + 100 + 1000 + 100000 + 1000000 + 10000000]$

$y = 0.0005 \text{ to } 0.0005 [1111110]$

$y = 5555.5555m$

c) Command window

```
clear
clc
syms x
syms y
x = (0:10);
y = 0.0005 to 0.0005 (x+(x^2)+(x.^3)+(x.^4)+(x.^5)+(x.^6)
   +(x.^7));
plot (x,y)
grid on
grid minor
xlabel('x')
ylabel('structural Deformation')
```

...

(y'))

x'(y').t

PIRAH O. SHEDRACK
17/ENG06/073
mechanical engineering.
ENG 381

solution

a) $x(x-1)y'' + (3x-1)y' + y = 0$

for the n^{th} derivative,

$$x(x-1)y^{n+2} + (2x-1)ny^{n+1} + n(n-1)y^n + (3x-1)y^{n+1} + 3ny^n + y^n = 0$$

$$x(x-1)y^{n+2} + y^{n+1}(2xn - n + 3x - 1) + y^n(n^2 - n + 3n + 1) = 0$$

at $x = 0$,

$$(y^{n+1})_0(n-1) + (y^n)_0(n^2+2n+1) = 0$$

$$-(n+1)(y^{n+1})_0 = -(n^2+2n+1)(y^n)_0$$

$$(n+1)(y^{n+1})_0 = (n^2+2n+1)(y^n)_0$$

$$(y^{n+1})_0 = (n+1)(y^n)_0$$

at $n = 0$

$$(y^{0+1})_0 = (0+1)(y^0)$$

$$(y')_0 = (y)_0$$

at $n = 1$

$$(y^{1+1})_0 = (1+1)(y')$$

$$(y'')_0 = 2(y')_0$$

at $n = 2$,

$$(y^{2+1})_0 = (2+1)(y^2)_0$$

$$(y^3)_0 = 3(y'')$$

$$(y^3)_0 = 6(y')_0$$

at $n = 3$

$$(y^4)_0 = (3+1)(y^3)_0$$

$$(y^4)_0 = 4(y^3)_0 = 4 \times 6(y')_0 = 24(y')_0$$

at $n = 4$.

$$(y^5)_0 = (4+1)(y^4)_0$$

$$= 5(y^4)$$

$$= 5 \times 24(y')_0$$

$$(y^5)_0 = 120(y')_0$$