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Petroleum Engineering

ENR 381 Assignment III

Equation 1

$$x(x-1)y'' + (3x-1)y' + y = 0$$

1st term = $x(x-1)y''$

$$u = y''$$

$$v = x(x-1)$$

$$u^n = y^{(n+2)}$$

$$v' = 2x-1$$

$$u^{(n-1)} = y^{(n+1)}$$

$$v'' = 2$$

$$u^{(n-2)} = y^n$$

$$v''' = 0$$

$$y^{(n+2)}(x^2-x) + n y^{(n+1)}(2x-1) + \frac{n(n-1)}{2!} y^n \cdot 2 + 0$$

2nd term = $(3x-1)y'$

$$u = y'$$

$$v = (3x-1)$$

$$u^n = y^{(n+1)}$$

$$v' = 3$$

$$u^{(n-1)} = y^n$$

$$v'' = 0, y^{(n+1)}(3x-1) + n y^n \cdot 3 + 0$$

3rd term = y

$$u = y$$

$$v = 1$$

$$u^n = y^n$$

$$v' = 0, y^n + 0$$

Adding the terms together.

$$y^n - \frac{u^n v}{2!} + n \frac{u^{(n-1)} v'}{2!} + n \frac{(n-1) u^{(n-2)} v''}{2!} + \frac{n(n-1)(n-2) u^{(n-3)} v'''}{3!} + \dots$$

$$y^{(n+2)}(x^2-x) + n y^{(n+1)}(2x-1) + \frac{n(n-1)}{2!} y^n \cdot 2 + y^{(n+1)}(3x-1) + n y^n \cdot 3 + y^n = 0$$

$$(x^2-x)y^{(n+2)} + (2xn-n)y^{(n+1)} + (3x-1)y^{(n+1)} + (n^2-n)y^n + 3ny^n + y^n = 0$$

$$(x^2-x)y^{(n+2)} + (2xn-n+3x-1)y^{(n+1)} + (n^2-n+3n+1)y^n = 0$$

$$x(x-1)y^{(n+2)} + (2xn+3n-n-1)y^{(n+1)} + (n^2+2n+1)y^n = 0$$

when $x=0$

$$0(0-1)y^{(n+2)} + (2(0)n+3n-n-1)y^{(n+1)} + (n^2+2n+1)y^n = 0$$

$$(-n-1)y^{(n+1)} + (n^2+2n+1)y^n = 0$$

$$-(n+1)(y^{(n+1)})_0 + (n^2+2n+1)(y^n)_0 = 0$$

$$-(n+1)[y^{(n+1)}]_0 = -(n^2+2n+1)[y^n]_0$$

$$[y^{(n+1)}]_0 = \frac{-\cancel{(n+1)}(n+1)[y^n]_0}{+\cancel{(n+1)}}$$

$$[y^{(n+1)}]_0 = (n+1)[y^n]_0$$

when $n=0$

$$[y^{(0+1)}]_0 = [0+1][y^0]_0$$

$$[y^{(1)}]_0 = [y^0]_0$$

when $n=1$

$$[y^{(1+1)}]_0 = [1+1][y^{(1)}]_0$$

$$[y^{(2)}]_0 = 2[y^{(1)}]_0$$

where $n=2$

$$[y^{(2+1)}]_0 = [2+1][y^{(2)}]_0$$

$$= 3[y^{(2)}]_0 = 3 \cdot 2[y^{(1)}]_0$$

$$= 6[y^{(1)}]_0$$

when $n=3$

$$[y^{(3+1)}]_0 = [3+1][y^{(3)}]_0$$

$$= 4[y^{(3)}]_0 = 4 \cdot 6[y^{(2)}]_0$$

$$= 24[y^{(2)}]_0$$

when $n=4$

$$[y^{(4+1)}]_0 = [4+1][y^{(4)}]_0$$

$$= 5[y^{(4)}]_0 = 5 \cdot 24[y^{(3)}]_0$$

$$= 120[y^{(3)}]_0$$

when $n=5$

$$[y^{(5+1)}]_0 = [5+1][y^{(5)}]_0$$

$$= 6[y^{(5)}]_0 = 6 \cdot 120[y^{(4)}]_0$$

$$= 720[y^{(4)}]_0$$

when $n=6$

$$[y^{(6+1)}]_0 = [6+1][y^{(6)}]_0$$

$$= 7[y^{(6)}]_0 = 7 \cdot 720[y^{(5)}]_0$$

$$= 5040[y^{(5)}]_0$$

$$y = (y^0)_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^{(4)})_0 + \frac{x^5}{5!}(y^{(5)})_0 + \frac{x^6}{6!}(y^{(6)})_0$$

$$y = (y^0)_0 + x (y^1)_0 + \frac{x^2}{2!} [2(y^2)_0] + \frac{x^3}{3!} [6(y^3)_0] + \frac{x^4}{4!} [24(y^4)_0] + \frac{x^5}{5!} [120(y^5)_0] + \frac{x^6}{6!} [720(y^6)_0] + \frac{x^7}{7!} [5040(y^7)_0]$$

$$y = [y^0]_0 + x [y^1]_0 + x^2 [y^2]_0 + x^3 [y^3]_0 + x^4 [y^4]_0 + x^5 [y^5]_0 + x^6 [y^6]_0 + x^7 [y^7]_0$$

$$y = (1+x)(y^0)_0 + (x^2+x^3+x^4+x^5+x^6+x^7)(y^1)_0$$

when $y_0 = 0.0005m$, $y'(0) = 0.0005$

b. $y = (1+x)(0.0005m) + (x^2+x^3+x^4+x^5+x^6+x^7)(0.0005)$
 when $x = 5m$

$$y = (1+5)(0.0005) + (5^2+5^3+5^4+5^5+5^6+5^7)(0.0005)$$

$$y = 0.003 + 97650(0.0005)$$

$$y = 48.828m$$

when $x = 8m$

$$y = (1+8)(0.0005) + (8^2+8^3+8^4+8^5+8^6+8^7)(0.0005)$$

$$= 0.0045 + 2396736(0.0005)$$

$$= 1198.3725m$$

when $x = 10m$

$$y = (1+10)(0.0005) + (10^2+10^3+10^4+10^5+10^6+10^7)0.0005$$

$$= 0.0055 + 1111100(0.0005)m$$

$$y = 555.5555m$$

c. Command window

clear

clc

close all

Syms x

$$x = 0:0.01:10$$

$$y = (1+x) * (0.0005) + ((x.^2+x.^3+x.^4+x.^5+x.^6+x.^7) * (0.0005))$$

plot(x,y)

grid on

