26/10/19

1) ~~Ax~~ $y_1 = 3e^{2x}$

$y_2 = 3e^{-x}$

$A = \int_1^2 y_1\,dx - \int_1^2 y_2\,dx$

$\int_1^2 y_1\,dx = \int_1^2 3e^{2x}\,dx$

$= 3\int_1^2 e^{2x}\,dx$

$= 3\left[\frac{1}{2}e^{2x}\right]_1^2$

$= \frac{3}{2}\left[e^{2x}\right]_1^2$

$= \frac{3}{2}\left[e^{2(2)} - e^{2(1)}\right]$

$= \frac{3}{2}\left[e^4 - e^1\right]$

$= \frac{3}{2}(47.2091$

$= 70.814$

$\int_1^2 y_2\,dx = \int_1^2 3e^{-x}\,dx$

$= 3\int_1^2 e^{-x}\,dx$

$= 3\left[-e^{-x}\right]_1^2$

$= 3\left[-e^{-2} - (-e^{-1})\right]$

$= 3(-e^{-2} + e^{-1})$

$= 3(0.2325)$

$= 0.6976$

$A = 70.814 - 0.6976$

$= 70.1164$ units^2

2) $y = 2\sin\frac{\pi}{10}t$

$x = 2t\ \ 2t - 2\cos\frac{\pi}{10}t$

$\int_0^{10} y\,dx = \int_0^{10} 2\sin\frac{\pi}{10}t\,dx$

$\frac{dx}{dt} = \cancel{2} + \frac{\pi}{5}\sin\frac{\pi}{10}t$

$dx = \cancel{2} + \frac{\pi}{5}\sin\frac{\pi}{10}t\,dt$

$\int_0^{10} y\,dx = \int_0^{10}\left(2\sin\frac{\pi}{10}t\right)\left(\cancel{2} + \frac{\pi}{5}\sin\frac{\pi}{10}t\right)dt$

$= \int_0^{10}\left[4\sin\frac{\pi t}{10} + \frac{2\pi}{5}\sin^2\frac{\pi t}{10}\right)dt$

$= \int_0^{10} 4\sin\frac{\pi t}{10}\,dt + \int_0^{10}\frac{2\pi}{5}\sin^2\frac{\pi t}{10}\,dt$

$= \int_0^{10} 4\sin\frac{\pi t}{10} + \int_0^{10}\frac{2\pi}{5}\,\frac{1-\cos\frac{2\pi t}{10}}{2}$

$= 4\int_0^{10}\sin\frac{\pi t}{10} + \frac{1}{2}\cdot\frac{2\pi}{5}\int_0^{10} 1-\cos\frac{2\pi t}{10}$

$= 4\left[\frac{-10}{\pi}\cos\frac{\pi t}{10}\right]_0^{10} + \frac{\pi}{5}\left[t - \frac{5}{\pi}\sin\frac{\pi t}{5}\right]_0^{10}$

$= \frac{-40}{\pi}\left[\frac{\cos\pi(10)}{10} - \frac{\cos\pi(0)}{10}\right] + \frac{\pi}{5}\left[\left(10 - \frac{5}{\pi}\sin\frac{\pi(10)}{5}\right) - \left(0 - \frac{5}{\pi}\sin\frac{\pi(0)}{5}\right)\right]$

$= \left[\frac{-40}{\pi}\cos\pi + 2\pi - \frac{5}{\pi}\sin 2\pi\right] - \left[\frac{-40}{\pi}\cos 0 +\right]$

$= \frac{40}{\pi} + 2\pi + \frac{40}{\pi}$

$= \frac{80}{\pi} + 2\pi$

$= 31.748$ units^2