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300level

ENG381: Engineering mathematics III

→ The Model for the deformation (y) of a structural is represented by the expression given in eqn ①

$$x(x-1)y'' + (3x-1)y' + y = 0 \quad \text{—— ①}$$

Given that $y(0) = 0.0005m$ and $y'(0) = 0.0005m$, applying Leibnitz-maclaurin method

ⓐ obtain the power series solution of the model up and including the term in x^7

ⓑ estimate the approximate deformation when $x = 5, 8$ and $10m$ and

ⓒ with the aid of a MATLAB mfile program, plot the response of the structure element for $0 \le x \le 10m$.

Soln

$$x(x-1)y'' + (3x-1)y' + y = 0$$

$$w_1 = x(x-1)y'', \quad w_2 = (3x-1)y'; \quad w_3 = y$$

$$\Rightarrow w_1 = x(x-1)y''$$

$x(x-1) = V$, $U = y''$ or $y^{(2)}$

$U^n = y^{n+2}$ | $V = x^2 - x$

$U^{n-1} = y^{n+1}$ | $V^{(1)} = 2x - 1$

$U^{n-2} = y^n$ | $V^{(2)} = 2$

$U^{n-3} = y^{n-1}$ | $V^{(3)} = 0$

$$\Rightarrow w_2 = (3x-1)y'$$

$U = y^{(1)}$ | $V = 3x - 1$

$U^n = y^{n+1}$ | $V^{(1)} = 3$

$U^{n-1} = y^n$ | $V^{(2)} = 0$

$$\Rightarrow w_3 = y, \quad U = y, \quad V = 1$$

$U^n = y^n$ | $V^{(1)} = 0$

∴ using $UV = U^nV + nU^{n-1}V^{(1)} + \frac{n(n-1)}{2!}U^{n-2}V^{(2)} + \frac{n(n-1)(n-2)}{3!}U^{n-3}V^{(3)}$ ---

$$\therefore w_1 = y^{n+2}\cdot(x^2 - x) + n\cdot y^{n+1}\cdot(2x-1) + \frac{n(n-1)}{2!}\cdot y^n \cdot 2$$

$w_2 =$

$$W_1 = y^{n+1} \cdot (3x-1) + n \cdot y^n \cdot 3$$

$$W_2 = y^n$$

Hence $\therefore W_1 + W_2 + W_3 = 0$

$$\Rightarrow y^{n+2} \cdot \cancel{(x^2 - x)} + n \cdot y^{n+1} \cdot (2x-1) + \frac{n(n-1)}{\cancel{2!}} \cdot y^n \cdot \cancel{2} + y^{n+1} \cdot (3x-1) + n \cdot y^n \cdot 3 + y^n = 0$$

assuming $x = 0$

$$\therefore n \cdot (y^{n+1})_0(-1) + n(n-1) \cdot (y^n)_0 + (y^{n+1})_0(-1) + n(y^n)_0 3 + (y^n)_0 = 0$$

$$-n \cdot (y^{n+1})_0 + (n^2 - n) \cdot (y^n)_0 - y^{n+1} + 3n \cdot y^n + (y^n)_0 = 0$$

$$-n(y^{n+1})_0 - (y^{n+1})_0 + (n^2 - n)(y^n)_0 + 3n \cdot (y^n)_0 + (y^n)_0 = 0$$

$$(y^{n+1})_0(-n-1) + (y^n)_0[(n^2 - n) + 3n + 1] = 0$$

$$(y^{n+1})_0(-n-1) + (y^n)_0(n^2 - n + 3n + 1) = 0$$

$$(y^{n+1})_0(-n-1) + (y^n)_0(n^2 + 2n + 1) = 0$$

$$(y^{n+1})_0(-n-1) = -(n^2 + 2n + 1) y^n$$

$$(y^{n+1})_0 = \frac{\cancel{-}(n^2 + 2n + 1)(y^n)_0}{\cancel{-}(n+1)}$$

$$y^{n+1} = \frac{(n^2 + 2n + 1)(y^n)_0}{n+1} \quad \text{— Recurrence Relation}$$

$\Rightarrow$ recall : $(y^{(0)})_0 = 0.0005m$

$(y^{(1)})_0 = 0.0005m$

$\Rightarrow$ when $n = 1$

$$\Rightarrow (y^{1+1})_0 = \frac{(1^2 + 2(1) + 1)}{1+1} (y^{(1)})_0$$

$$(y^{(2)})_0 = 2(y^{(1)})_0 = 2 \times 0.0005 = 1 \times 10^{-3} m //$$

$\Rightarrow$ when $n = 2$

$$\Rightarrow (y^{2+1})_0 = \frac{(2^2 + 2(2) + 1)}{2+1} (y^{(2)})_0$$

$$(y^{(3)})_0 = 3(y^{(2)})_0 = 3 \times 1 \times 10^{-3} = 3 \times 10^{-3} m //$$

$\Rightarrow$ when $n=3$

$$\Rightarrow \left(y^{(3+1)}\right)_0 = \frac{(3^2+2(3)+1)}{3+1}\left(y^{(3)}\right)_0 \Rightarrow \left(y^{(4)}\right)_0 = 4\left(y^{(3)}\right)_0$$

$$\left(y^{(4)}\right)_0 = 4\times 3\times 10^{-3} = 0.012m$$

$\Rightarrow$ when $n=4$

$$\Rightarrow \left(y^{(4+1)}\right)_0 = \frac{(4^2+2(4)+1)}{4+1}\left(y^{(4)}\right)_0 \Rightarrow$$

$$\left(y^{(5)}\right)_0 = 5\left(y^{(4)}\right)_0 = 5(0.012) = 0.06m$$

$\Rightarrow$ when $n=5$

$$\left(y^{(5+1)}\right)_0 = \frac{(5^2+2(5)+1)}{5+1}\left(y^{(5)}\right)_0$$

$$\left(y^{(6)}\right)_0 = 6\left(y^{(5)}\right)_0 = 6\times 0.06 = 0.36m$$

$\Rightarrow$ when $n=6$

$$\left(y^{(6+1)}\right)_0 = \frac{(6^2+2(6)+1)}{6+1} = \left(y^{(6)}\right)_0$$

$$\left(y^{7}\right)_0 = 7\left(y^{(6)}\right)_0 = 7\times 0.36 = 2.52m$$

$\Rightarrow$ applying Leibnitz-maclaurin's Theorem

using $(y)_0 = \left(y^{(0)}\right)_0 + x\left(y^{(1)}\right)_0 + \frac{x^2}{2!}\left(y^{(2)}\right)_0 + \frac{x^3}{3!}\left(y^{(3)}\right)_0 + \frac{x^4}{4!}\left(y^{(4)}\right)_0 + \cdots$

$$(y)_0 = 0.0005 + x\times 0.0005 + \frac{x^2}{2!}\cdot 1\times 10^{-3} + \frac{x^3}{3!}\times 3\times 10^{-3} + \frac{x^4}{4!}\cdot 0.012 +$$

$$\frac{x^5}{5!}\times 0.06 + \frac{x^6}{6!}\cdot 0.36 + \frac{x^7}{7!}\cdot 2.52$$

$$(y)_0 = 0.0005 + 0.0005x + 0.0005x^2 + 0.0005x^3 + 0.0005x^4 + \cdots$$
$$+ 0.0005x^5 + 0.0005x^6 + 0.0005x^7 + \cdots$$

a) $(y)_0 = 0.0005(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$

b) When $x=5$

$\therefore (y)_0 = 0.0005(1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)$

$$(y)_0 = 48.828m$$

→ when $x = 8$

$\therefore (y)_o = 0.0005(1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$

$(y)_o = 1198.3685 m$

→ when $x = 10$

$\therefore (y_o)_o = 0.0005(1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$

$(y)_o = 5555.5555 m$

(c)

Command window

```
clear
close all
clc
syms x
y = 0.0005*(1+x+x^2+x^3+x^4+x^5+x^6+x^7)
x = 0:1:10
yp = subs(y)
ypp = double(yp)
plot(x, ypp)
legend('y(m)')
grid on
grid minor
```

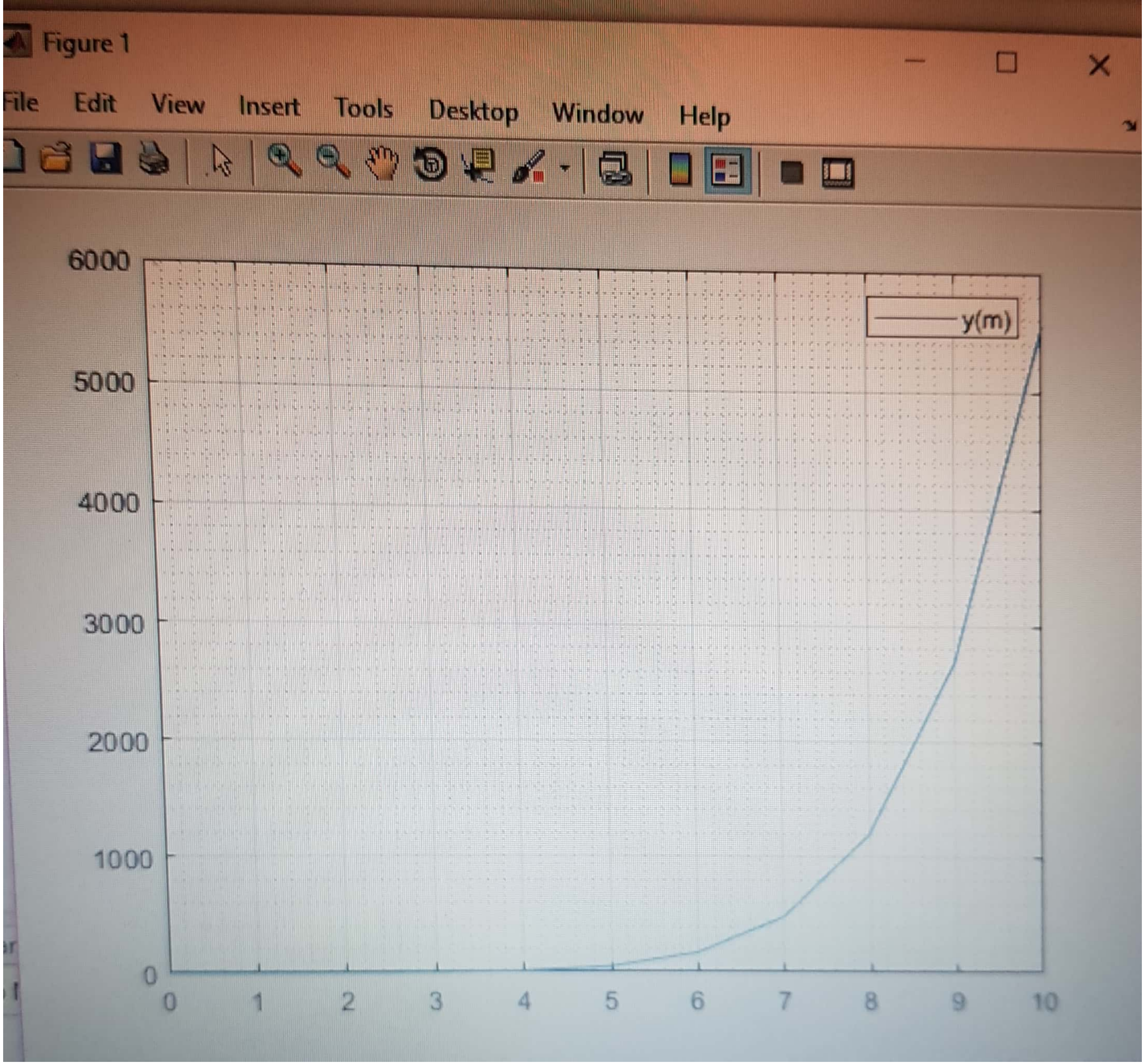
Figure 1
File Edit View Insert Tools Desktop Window Help
y(m)
6000
5000
4000
3000
2000
1000
0
0
1
2
3
4
5
6
7
8
9
10