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BIOMEDICAL ENGINEERING
17/ENG08/004
ENG 384 ⇒ ENGINEERING MATHEMATICS III
ASSIGNMENT III

Question.

$$x(x-1)y'' + (3x-1)y' + y = 0.$$
$$(x^2-x)y'' + (3x-1)y' + y = 0$$
$$A + B + C = 0.$$

LEIBNITZ - MACLAURIN METHOD.

$$y = U^n V + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V'' + \frac{n(n-1)(n-2)}{3!}U^{n-3}V''' + \frac{n(n-1)(n-2)(n-3)}{4!}U^{n-4}V^4 + \cdots$$

$$A = (x^2-x)y''$$
$$U = y'' \Rightarrow U^n = y^{2+n}; \quad U^{n-1} = y^{1+n}; \quad U^{n-2} = y^n$$
$$V = x^2 - x \quad V' = 2x-1; \quad V'' = 2; \quad V''' = 0.$$

$$A^n = y^{n+2}(x^2-x) + n(y^{1+n})(2x-1) + \frac{n(n-1)}{2!}y^n \cdot 2 + 0$$

$$A^n = y^{n+2}(x^2-x) + n(y^{1+n})(2x-1) + (n^2-n)y^n + 0.$$

$B = (3x-1)y'$

$u = y' \; ; \; u^n = y^{1+n} \; ; \; u^{n+1} = y^n$

$v = 3x-1 \quad v' = 3 \quad v'' = 0.$

$B^n = y^{1+n}(3x-1) + 3n y^n + 0.$

$C = y$

$C^n = y^n.$

$A + B + C = 0$

$y^{n+2}(x^2-x) + n y^{1+n}(2x-1) + (n^2-n)y^n + y^{1+n}(3x-1) + 3n y^n + y^n = 0$

Collecting like terms.

$0 = y^{n+2}(x^2-x) + y^{1+n}(2nx-n+3x-1) + y^n(n^2-n+3n+1)$

$= 0 \cdot$ at $x = 0.$

$= (y^{n+2})_0(0^2-0) + (y^{1+n})_0(2n(0)-n+3(0)-1) + (y^n)_0(n^2+2n+1)$

$= (y^{1+n})_0(-n-1) + (y^n)_0(n^2+2n+1)$

$= -y^{1+n}(n+1) + (y^n)_0(n^2+2n+1).$

$(y^{1+n})_0(n+1) = (y^n)_0(n^2+2n+1)$

$(y^{1+n})_0 = \dfrac{(n^2+2n+1)y^n}{(n+1)}$

n^2+2n+1

$n^2 = n, n.$

$n^2+n+n+1$

$n(n+1)+1(n+1) = (n+1)(n+1)$

$$\therefore y^{1+n} = \frac{(n+1)(n+1)y^n}{(n+1)}$$

$$\boxed{y^{1+n} = (n+1)(y^n)_0} \rightarrow \text{RECURRENT EQUATION}$$

at $n = 0$.

$$(y^1)_0 = (y^0)_0$$

when $n = 1$

$$(y^{11})_0 = 2(y^1)_0$$

when $n = 2$

$$(y^{111})_0 = 3(y^{11})_0 = 6(y^1)_0$$

when $n = 3$

$$(y^4)_0 = 4(y^{111})_0 = 24(y^1)_0$$

when $n = 4$

$$(y^5)_0 = 5(y^4)_0 = 120(y^1)_0.$$

when $n = 5$

$$(y^6)_0 = 6(y^5)_0 = 720(y^1)_0.$$

when $n = 6$

$$(y^7)_0 = 7(y^6)_0 = 5040(y^1)_0$$

9. POWER SERIES.

$$y = (y^0)_0 + x(y^1)_0 + \frac{x^2}{2!}(y^{11})_0 + \frac{x^3}{3!}(y^{111})_0 + \frac{x^4}{4!}(y^4)_0 + \frac{x^5}{5!}(y^5)_0 + \frac{x^6}{6!}(y^6) + \frac{x^7}{7!}(y^7)_0 + \cdots$$

$$y = (y)_0 + x(y^0)_0 + \frac{x^2}{2!}(y^1)_0 \cdot 2 + \frac{x^3}{3!} \cdot 6(y^1)_0 + \frac{x^4}{4!} 24(y^1)_0 + \frac{x^5}{5!} \cdot 120(y^1)_0 + \frac{x^6}{6!} \cdot 720(y^1)_0 + \frac{x^7}{7!} \cdot 5040(y^1)_0$$

$$y = (y)_0(1+x) + (y')_0[x^2+x^3+x^4+x^5+x^6+x^7)$$

Given that $(y)_0 = 0.0005m$

$(y')_0 = 0.0005$

$$y = 0.0005m(1+x) + 0.0005(x^2+x^3+x^4+x^5+x^6+x^7)$$

b. When $x = 5m$, $8m$ and $10m$

$\therefore y = 0.0005m(1+5) + 0.0005(5^2+5^3+5^4+5^5+5^6+5^7)$

$y = 0.0005m(6) + 0.0005(25+125+625+3,125 + 15,625 + 78,125)$

$y = 0.0005m(6) + 0.0005(97650m)$

$y = 0.0005m(6) + 48.825m$

$y = 0.003m + 48.825m$

$y = 48.828m \simeq 49m$

When $x = 8m$.

$y = 0.0005m(1+8) + 0.0005(8^2+8^3+8^4+8^5+8^6+8^7)$

$y = 0.0005m(9) + 0.0005(64+512+4,096+32,768 + 262,144 + 2,097,152)$

$y = 0.0005m(9) + 0.0005(2,396,736)$

$= 0.0045 + 1,198.368m$

$y = 1198.3725m \simeq 1198m$

When $x = 10m$.

$y = 0.0005(1+10) + 0.0005(10^2+10^3+10^4+10^5+10^6+10^7)$

$y = 0.0005(11) + 0.0005(100+1000+10000+100,000+1000000 + 10,000,000)$

$Y = 0.0005m(11) + 0.0005(11,116,100m)$

$= 0.0055m + 5555.55m$

$Y = 5,555.5555m \approx 5,556m. \approx 5556m$

c

Command Window.

```
clear
clc
Close all.
Sym x
x = [(1+x)*(0.0005)] + [(x^2+x^3+x^4+x^5+x^6
                +x^7)*(0.0005)].
t = 0:0.01:10.
xt = Subs(x,t).
xtn = double(xt).
plot(t,xtn)
xlable('t')
ylable('x')
Grid on
Grid minor.
axis tight.
```