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17/ENG07/021

CHEMICAL ENGINEERING

ENG 381 Assignment III

sol

$$x(x-1)y'' + (3x-1)y' + y = 0$$

let

$kl_1 = x(x-1)y''$

$kl_2 = (3x-1)y'$

$kl_3 = y$

for kl_1

$u = y^2 \qquad v = x(x-1) = x^2 - x$

$u^n = y^{n+2} \qquad v' = 2x-1$

$u^{n-1} = y^{n+1} \qquad v'' = 2$

$u^{n-2} = y^n \qquad v''' = 0$

kl_2

$u = y' \qquad v = 3x-1$

$u^n = y^{n+1} \qquad v' = 3$

$u^{n-1} = y^n \qquad v'' = 0$

kl_3

$u = y \qquad v = 1$

$u^n = y^n \qquad v' = 0$

Recall

$$y = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3 + \cdots$$

$$y = y^{(n+2)} \cdot (x^2-x) + n(y^{(n+1)})(2x-1) + \frac{n(n-1)}{2!} y^n \cdot 2 + \frac{n(n-1)(n-2)}{3!}(y^{(n-1)}) \cdot 0 + (y^{(n+1)})(3x-1) +$$

$$n y^n \cdot 3 + \frac{n(n-1)}{2!}(y^{(n-1)}) \cdot 0 + y^n \cdot 1 + n y^{(n-1)} \cdot 0$$

$$= (x^2-x)(y^{(n+2)})_0 + n(2x-1)(y^{(n+1)}) + n(n-1)(y^{(n)}) + (3x-1)(y^{(n+1)})_0 + n3(y^n)_0 + (y^n)_0$$

$$= (x^2-x)(y^{(n+2)})_0 + (y^{(n+1)})_0(n(2x-1) + (3x-1)) + (y^{(n)})_0(n(n-1) + 3n + 1)$$

$$= (x^2-x) \cdot (y^{n+2})_0 + (y^{(n+1)})_0(2nx - n + 3x - 1) + (y^n)_0(n^2 - n + 3n + 1)$$

~~at~~ assuming $x = 0$

$$(y^{(n+1)})_0(-n-1) + (y^n)(n^2+2n+1) = 0$$

$-(y^{(n+1)})(n+1) + (y^{(n)})_0 (n^2+2n+1) = 0$

$(y^{(n+1)})_0 (n+1) = (y^{(n)})_0 (n^2+2n+1)$

$(y^{(n+1)})_0 = (y^{(n)})_0 \dfrac{(n+1)(n+1)}{(n+1)}$

$(y^{(n+1)})_0 = (y^{(n)})_0 (n+1).$

Recall, $(y^{(0)})_0 = 0.0005$

$(y^{(1)})_0 = 0.0005$

at $n=0$

$(y^{(0+1)})_0 = (y^0)(0+1)$

$(y^{(1)})_0 = 1.(y^0)_0$

$n=1$

$(y^{(1+1)})_0 = (y^1)(1+1)$

$(y^2)_0 = 2(y^{(1)})_0 = \cancel{2(y^0)_0}$

$n=2$

$(y^{(2+1)})_0 = (y^2)(2+1)$

$(y^3)_0 = 3(y^2) = 3\times 2(y^1) = 6(y^1)$

$n=3$

$(y^{(3+1)})_0 = (y^3)(3+1)$

$(y^4)_0 = 4(y^3) = 4\times 6(y^1)_0 = 24(y^1)$

$n=4$

$(y^{(4+1)})_0 = (y^4)(4+1)$

$(y^5)_0 = 5(y^4) = 5\times 24(y^1)_0 = 120(y^1)_0$

$n=5$

$(y^{(5+1)})_0 = (y^5)(5+1)$

$(y^6)_0 = 6(y^5) = 6\times 120(y^1) = 720(y^1)_0$

$n=6$

$(y^{(6+1)})_0 = (y^6)(6+1)$

$(y^7)_0 = 7(y^6) = 7\times 720(y^1)_0 = 5040(y^1)_0$

Using Leibnitz Maclaurin theory.

$y = (y^{(0)})_0 + x(y^1)_0 + \dfrac{x^2}{2!}(y^2)_0 + \dfrac{x^3}{3!}(y^3)_0 + \dfrac{x^4}{4!}(y^4)_0 + \dfrac{x^5}{5!}(y^5)_0 + \dfrac{x^6}{6!}(y^6) + \dfrac{x^7}{7!}(y^7)_0$

$$Y = (Y^{(0)})_0 + x(Y^{0})_0 + \frac{x^2}{2!}(2Y') + \frac{x^3}{3!}(6Y') + \frac{x^4}{4!}(24Y') + \frac{x^5}{5!}(120Y') + \frac{x^6}{6!}(720Y') + \cdots$$

$$\cdots + \frac{x^7}{7!}(5040\,Y')_0$$

$$Y = (Y^{(0)})_0 (1+x) + (x^2+x^3+x^4+x^5+x^6+x^7)(Y')$$

$$Y = 0.0005(1+x) + 0.0005(x^2+x^3+x^4+x^5+x^6+x^7)$$

when $x = 5$

$$Y = 0.0005(1+5) + 0.0005(5^2+5^3+5^4+5^5+5^6+5^7)$$

$$= 48.828\,m$$

$x = 8$

$$Y = 0.0005(1+8) + 0.0005(8^2+8^3+8^4+8^5+8^6+8^7)$$

$$= 1198.3725\,m$$

$x = 10$

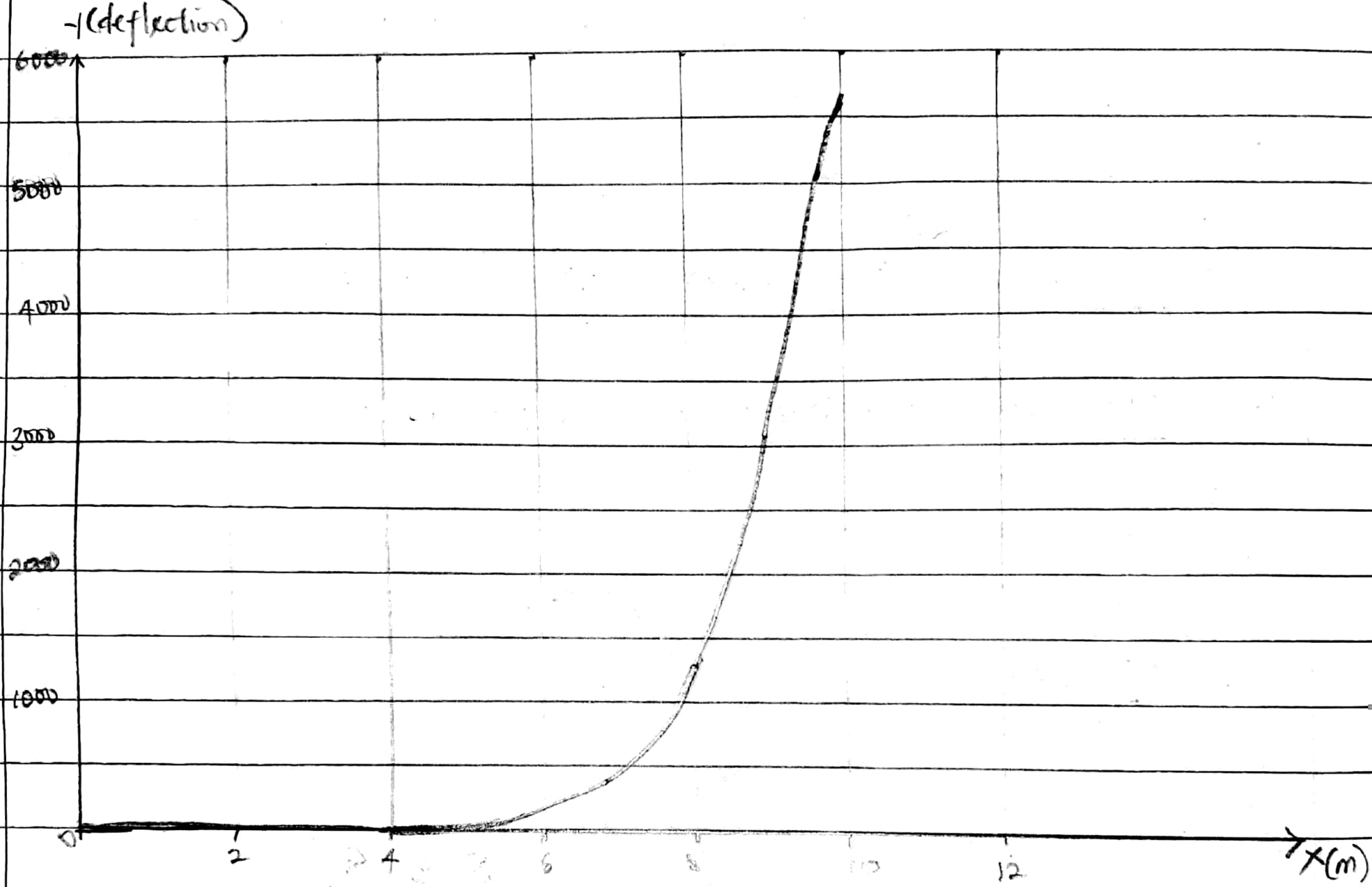
$$Y = 0.0005(1+10) + 0.0005(10^2+10^3+10^4+10^5+10^6+10^7)$$

$$= 5555.5555\,m.$$

Matlab file.

Commandwindow

```
Clear
Clc
Close all
x = 0:0.01:10
Y = (0.0005*(1+x)) + ((x^2+x^3+x^4+x^5+x^6+x^7)*0.0005)
Yn = Subs Y
Plot (x, Yn)
x label ('m')
Y label ('Deflection')
axis on
grid on
grid minor.
```



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