EJUOMEATSE TOSAN/ DORCAS.
17/ENG06/029
MECHANICAL ENGINEERING.

$x(x-1)y'' + (3x-1)y' + y = 0.$

Applying Leibnitz-Maclaurin method.

$(x^2 - x)y'' + (3x-1)y' + y = 0.$

$(x^2 - x)y^{(2)} + (3x-1)y' + y = 0.$

Find the nth derivative

Let $(x^2 - x)y^{(2)} = W_1$

$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \dots$

$u = y^2 \qquad v = x^2 - x.$

$u^n = y^{n+2} \qquad v' = 2x - 1$

$u^{n-1} = y^{n+1} \qquad v'' = 2.$

$u^{n-2} = y^n \qquad v''' = 0.$

$W_1^{(n)} = y^{n+2} \cdot x^2 - x + n y^{n+1} \cdot 2x - 1 + \frac{n(n-1)y^n \cdot 2}{2!}$

Let $(3x-1)y' = W_2.$

$u = y' \qquad v = 3x - 1$

$u^n = y^{n+1} \qquad v' = 3$

$u^{n-1} = y^n \qquad v'' = 0.$

$W_2^n = y^{n+1} \cdot 3x - 1 + n y^n \cdot 3.$

Let $y = W_3$

$W_3^{(n)} = y^n.$

$\therefore$ nth derivative

$y^{n+2} \cdot (x^2 - x) + n y^{n+1} \cdot 2x - 1 + n(n-1)y^n + y^{n+1} 3x + $
$+ n y^n \cdot 3 + y^n = 0$

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Assuming $x=0$.

$n(n-1)y^n + 3ny^n + y^n - ny^{n+1} - y^{n+1} = 0.$

$(n^2+2n+1)y^n - (n+1)y^{(n+1)} = 0.$

$y^{n+1} = \frac{n^2+2n+1}{n+1} y^n = \frac{(n+1)\cancel{(n+1)}y^n}{\cancel{(n+1)}}$

$y^{n+1} = (n+1)y^n$ → Recurrence relation

when $n=0$ $(y')_0 = (y'')_0$.

$n=1$ $(y^2)_0 = 2(y')_0 = (2y^0)_0.$

$n=2$ $(y^3)_0 = 3(y^2)_0 = 3\times 2(y^0)_0$
$= 6(y^0)_0.$

$n=3$ $(y^4)_0 = 4(y^3)_0 = 4\times 6(y^0)_0$
$= 24(y^0)_0$

$n=4$ $(y^5)_0 = 5(y^4)_0 = 5\times 24(y^0)_0$
$= 120(y^0)_0.$

$n=5$ $(y^6)_0 = 6(y^5)_0 = 6\times 120(y^0)_0$
$= 720(y^0)_0$

$n=6$ $(y^7)_0 = 7(y^6)_0 = 7\times 720(y^0)_0$
$= 5040(y^0)_0.$

Mauclaunn Series

$y = (y^0)_0 + x(y'')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3) \ldots$

$y = (y^0)_0 + x(y^0)_0 + \frac{x^2}{2!}(2y^0)_0 + \frac{x^3}{3!}(6y^0)_0 + \ldots$

$+\frac{x^4}{4!}(24(y^0)_0 + \frac{x^5}{5!}(120(y^0)_0 + \frac{x^6}{6!}(720(y^0)_0$

$+\frac{x^7}{7!}(5040(y^0)_0) + \ldots\ldots$

$y = (y^0)_0 + x(y^0)_0 + x^2(y^0)_0 + x^3(y^0)_0 + x^4(y^0)_0 +$
$x^5(y^0)_0 + x^6(y^0)_0 + x^7(y^0)_0 + \ldots$

a) Power Series

$y = (y^0)_0 (1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$.

Since $(y^0)_0 = 0.0005m$ and
$(y^{(1)}) = 0.0005$

Power series: $y = 0.0005 (1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$

b) When $x = 5m$

$y = 0.0005(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 \ldots)$

$y = 0.0005(1 + 5 + 5^2 + 5^3 + 5^4 + 5^6 + 5^7)$

$y = 48.828m$

When $x = 8m$

$y = 0.0005(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 \ldots)$

$y = 0.0005(1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$

$y = 1198.3725m$

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When $x = 10m$

$y = 0.0005\,(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 \ldots)$

$y = 0.0005\,(1 \times 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$

$y = 5555.5555\,m$

C MATLAB mfile

* Commandwindow
* Clear
* Clc
* close all
* x = 0:0.01:10
* y = 0.005 * (1+x+x^2+x^3+x^4+ x^5+x^6+x^7)
* yn = subs(y)
* Plot (x, yn)
* xlabel ('m')
* ylabel ('Deflection')
* axis tight
* grid on
* grid minor