

Ugah-Uche Chukwuraenyi

17/ENGG04/071

ELECT/ELECT

~~Area~~

Applying Leibnitz method:

$$x(x-1)y'' + (3x-1)y' + y = 0$$

For $G_1 - x(x-1)y''$

$$u = y'' \quad , \quad v = x^2 - x$$

$$u' = y''' \quad \quad v' = 2x - 1$$

$$u'' = y^{(4)} \quad \quad v'' = 2$$

$$v''' = 0$$

$$u^n v + n u^{n-1} v' + \frac{n(n-1)}{2} u^{n-2} v'' + \frac{n(n-1)(n-2)}{6} u^{n-3} v'''$$

$$(y^{n+2})(x^2-x) + n(y^{n+1})(2x-1) + \frac{n(n-1)}{2}(y^n) \\ x^2-x(y^{n+2}) + 2xn - n(y^{n+1}) + \frac{n^2-n}{2}(y^n)$$

For $G_2 - (3x-1)y'$

$$v = 3x-1$$

$$u = y'$$

$$v' = 3$$

$$u'' = y^{(3)}$$

$$v'' = 0$$

$$(y^{n+1})(3x-1) + n(y^n)(3)$$

for G_3

$$u^n = y^n \quad , \quad v = 1$$

$$y^n$$

Combining

$$x^2-x(y^{n+2}) + n(2x-1)(y^{n+1}) + n(n-1)(y^n) + (3x-1)(y^{n+1}) + 3n(y^n) + y^n$$

Solving

$$\Rightarrow x^2-x(y^{n+2}) + (y^{n+1})(n(2x-1) + (3x-1)) + (n(n-1) + 3n+1)y^n$$

$$\Rightarrow x^2-x(y^{n+2}) + (n(2x-1) + 3x-1)y^{n+1} + (n^2+2n+1)y^n$$

$$x^2-x(y^{n+2}) + (n(2x-1) + (3x-1))y^{n+1} + (n^2+2n+1)y^n = 0$$

at $x=0$

$$(n(n-1) + (-1))y^{n+1} + (n^2+2n+1)y^n = 0$$

$$(-n-1)y^{n+1} + (n^2+2n+1)y^n = 0$$

$$(y^{n+1})_0 = \frac{-(n^2 + 2n + 1) \cdot y^n}{n-1}$$

$$= \frac{-(n^2 + 2n + 1) y^n}{n+1}$$

$$(y^{n+1})_0 = \frac{n^2 + 2n + 1}{n+1} (y^n)_0$$

$$y^{n+1} = n+1 y^n$$

at $n=0$

$$y'_0 = 1 y_0$$

at $n=1$

$$y''_0 = 2 y'_0 = 2$$

$n=2$

$$y'''_0 = 3 y''_0 = 3(2)(y'_0) = 6 y'_0$$

$n=3$

$$y^{(4)}_0 = 4 y'''_0 = 4(3)(2)(y'_0) = 24 y'_0$$

$n=4$

$$y^{(5)}_0 = 5 y^{(4)}_0 = (2)(3)(4)(5)(y'_0)$$

$n=5$

$$y^{(6)}_0 = 6 y^{(5)}_0 = (2)(3)(4)(5)(y'_0) = (2)(3)(4)(5)(6)$$

$n=6$

$$y^{(7)}_0 = 7 y^{(6)}_0 = (2)(3)(4)(5)(6)(y'_0) = (2)(3)(4)(5)(6)(7)$$

Maclaurin series:

$$y = y_0 + x(y'_0) + \frac{x^2}{2!} (y''_0) + \frac{x^3}{3!} (y'''_0) + \frac{x^4}{4!} (y^{(4)}_0) + \frac{x^5}{5!} (y^{(5)}_0) +$$

$$\frac{x^6}{6!} (y^{(6)}_0) + \frac{x^7}{7!} (y^{(7)}_0)$$

$$y = (y_0) + x(y'_0) + \frac{x^2}{2!} (2y'_0) + \frac{x^3}{3!} (3! y'_0) + \frac{x^4}{4!} (4! y'_0) +$$

$$\frac{x^5}{5!} (5! y'_0) + \frac{x^6}{6!} (6! y'_0) + \frac{x^7}{7!} (7! y'_0)$$

$$y = y_0 + y'_0 x + y'_0 x^2 + y'_0 x^3 + x^4 y'_0 + x^5 y'_0 + x^6 y'_0 + x^7 y'_0$$

$$y = y_0 + y'(x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

Recall

$$y_1 = y_0$$

$$y_0 [1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7]$$

(b) $y'(0) = 0.0005$

when $x = 5$

$$\begin{aligned} y_5 &= 0.0005 [1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7] \\ &= 0.0005 (97656) \\ &= 48.828 = \underline{49} \end{aligned}$$

when $x = 6$, $y_0 = 0.0005$

$$\begin{aligned} y_6 &= 0.0005 [1 + 6 + 6^2 + 6^3 + 6^4 + 6^5 + 6^6 + 6^7] \\ &= 0.0005 (2396745) \\ &= 1198.3725 = \underline{1198} \end{aligned}$$

$x = 10m$

$$\begin{aligned} y_{10} &= 0.0005 (1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7) \\ &= 0.0005 (11111111) \\ &= 5555.5555 \\ &= \underline{5556} \end{aligned}$$

(c) Command Window:

clc

clear all

close all

syms x y

$x = 0:0.1:10$

$y = (0.0005) * (1 + x + x.^2 + x.^3 + x.^4 + x.^5 + x.^6 + x.^7)$

$y_n = \text{subs}(y)$

$y_{nn} = \text{double}(y_n)$

plot(x, y_{nn})

x label('x')

y label('T')

Grid on

Grid minor

Axis right

$[x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7 \ x_8 \ x_9 \ x_{10}]$

$2000.0 - 1000$

$2 - 1000$

$[2 - 1000] + [2 - 1000] + [2 - 1000] + [2 - 1000] + [2 - 1000]$

$(2 - 1000) + (2 - 1000)$

