OMENIOKU PERPETUAL 26/10/19.

18/ENG06/060

MECHANICAL

1. Find the area bounded by the curves $y = 3e^{2x}$ and $y = 3e^{-x}$ and the ordinates at $x = 1$ and $x = 2$

2. The parametric equations of a curve are $y = 2\sin\frac{\pi}{10}t$ and $x = 2 + 2t - 2\cos\frac{\pi}{10}t$
Find the area under the curve between $t = 0$ and $t = 10$

ANSWERS

1. $y = 3e^{2x}$

$y = 3e^{-x}$

$$A = \int_{x_1}^{x_2} f(x) - g(x)\, dx$$

$$= \int_1^2 3e^{2x} - 3e^{-x}\, dx$$

$$= 3\int_1^2 e^{2x} - e^{-x}\, dx$$

$$= 3\left|\frac{e^{2x}}{2} - e^{-x}\right|_1^2$$

$$= 3\left[\frac{e^{2(2)}}{2} + e^{(-2)}\right] - 3\left[\frac{e^{2(1)}}{2} + e^{(-1)}\right]$$

$$= (81.9 + 0.41) - (11.08 + 1.10)$$

$$= 82.31 - 12.18$$

$$= 70.13 \text{ square unit}$$

1. $y = 2\sin\frac{\pi}{10}t$

$x = 2+2t - 2\cos\frac{\pi}{10}t$

$$A = \int_{t_1}^{t_2} y(t)\cdot\frac{dx}{dt}\cdot dt$$

$$A = \int_0^{10} 2\sin\frac{\pi}{10}t \times 2 + \frac{2\pi}{5}\sin\frac{\pi}{10}\,dt$$

$$A = 4\int_0^{10} \left(\sin\frac{\pi}{10}t\right)\left(1+\frac{2\pi}{5}\sin\frac{\pi}{10}t\right)dt$$

$$A = 4\int_0^{10} \left(\sin\frac{\pi}{10}t\right)\left(1+{}^{2\pi}/_{5}\sin{}^{\pi}/_{10}t\right)dt$$

$$A = 4\int_0^{10} \left(\sin\frac{\pi}{10}t + \frac{2\pi}{5}\sin^2\frac{\pi}{10}t\right)dt$$

Recall $\sin^2\frac{\pi}{10}t = \frac{1-\cos{}^{\pi}/_{5}t}{2}$

$$A = 4\int_0^{10} \sin\frac{\pi}{10} + \frac{2\pi}{5}\left[\frac{1-\cos{}^{\pi}/_{5}t}{2}\right]dt$$

Integrationg by parts

$$A = 4\int_0^{10} \sin\frac{\pi}{10}\,dt + \frac{2\pi}{10}\int_0^{10} 1-\cos\frac{\pi}{5}t\,dt$$

$$A = 4\int -\frac{10}{\pi}\cos\frac{\pi}{10}t\Big|_0^{10} + \frac{\pi}{5}\left[t-\frac{5}{\pi}\sin\frac{\pi}{5}t\right|_0^{10}$$

$$A = \left[4\left(-\frac{10}{\pi}\cos\frac{\pi}{10}(10) + \frac{\pi}{5}\left(10-\frac{5}{\pi}\sin\frac{\pi}{5}(10)\right]\right.\right. - \left[4\left(-\frac{10}{\pi}\cos\frac{\pi}{10}(0) + \frac{\pi}{5}\left(0-\frac{5}{\pi}\sin\frac{\pi}{5}(0)\right)\right.\right.$$

$$A = (12.73+2\pi) - (-12.73+0)$$

$= 31.744$ square unit