b) when $y_0 = 0.0005m$, $y'(0) = 0.0005$

$y = (1+x)(0.0005m) + (x^2+x^3+x^4+x^5+x^6+x^7)(0.0005)$

when $x = 5m$

$y = (1+5)(0.0005) + (5^2+5^3+5^4+5^5+5^6+5^7)(0.0005)$

$y = 0.0[illegible]3 + 97650(0.0005)$

$y = 48.828m$

when $x = 8m$

$y = (1+8)(0.0005) + (8^2+8^3+8^4+8^5+8^6+8^7)(0.0005)$

$y = 0.0045 + 2396736(0.0005)$

$y = 1198.3725m$

when $x = 10m$

$y = (1+10)(0.0005) + (10^2+10^3+10^4+10^5+10^6+10^7)(0.0005)$

$= 0.0055 + 11111100(0.0005)m$

$y = 5555.5555m$

c) Command window

```
clear
clc
close all
syms x
x=0:0.01:10
y=(1+x)*(0.0005)+((x.^2+x.^3+x.^4+x.^5+x.^6+x.^7)*...
    (0.0005))
plot(x,y)
 grid on
 grid minor
```

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ENG 381 Assignment III

The model for the deformation (y) of a structural element is represented by the expression given in equation (1):

$x(x-1)y'' + (3x-1)y' + y = 0$ —— 1

Given that $y(0) = 0.005m$ and $y'(0) = 0.0005$, applying Leibnitz-Maclaurin method

a) obtain the power series solution of the model up to and including the term in x^7

b) estimate the approximate deformation when x = 5, 8 and 10m and

c) with the aid of a MATLAB m-file program, plot the response of the structural element for $0 \le x \le 10m$

Solution

a) $x(x-1)y'' + (3x-1)y' + y = 0$ —— 1

$\omega_1 = x(x-1)y''$

$u = y''$ $\quad v = x(x-1) = x^2 - x$

$u^n = y^{(n+2)}$ $\quad v' = 2x-1$

$u^{(n-1)} = y^{(n+1)}$ $\quad v'' = 2$

$u^{(n-2)} = y^n$ $\quad v''' = 0$

$\omega_2 = (3x-1)y'$

$u = y'$ $\quad v = (3x-1)$

$u^n = y^{(n+1)}$ $\quad v' = 3$

$u^{(n-1)} = y^n$ $\quad v'' = 0$

$\omega_3 = y$

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v' = 0$

$\omega_1 = y^{(n+2)}(x^2 - x) + n(y^{(n+1)})(2x-1) + \frac{n(n-1)}{2!} y^n \cdot 2 + 0$

$w_2 = y^{(n+1)}(3x-1) + ny^{n} \cdot 3 + 0$

$w_3 = y^{n} + 0$

Adding the three terms together

$$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v'' + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v''' + \dots$$

$$= y^{(n+2)}(x^2 - x) + n y^{(n+1)} \cdot (2x-1) + \frac{n(n-1)}{2!} y^{n} \cdot 2 + y^{(n+1)}(3x-1) + n y^{n} \cdot 3 + y^{n} =$$

$$(x^2 - x) y^{(n+2)} + (2xn - n) y^{(n+1)} + (3x-1) y^{(n+1)} + (n^2 - n) y^{n} + 3n y^{n} + y^{n} = 0$$

$$= (x^2 - x) y^{(n+2)} + (2xn - n + 3x - 1) y^{(n+1)} + (n^2 - n + 3n + 1) y^{n} = 0$$

$$= x(x-1) y^{(n+2)} + (2xn + 3x - n - 1) y^{(n+1)} + (n^2 + 2n + 1) y^{n} = 0$$

when $x = 0$

$$= 0(0-1) y^{(n+2)} + (2(0)n + 3(0) - n - 1) y^{(n+1)} + (n^2 + 2n + 1) y^{n} = 0$$

$$= (-n-1) y^{(n+1)} + (n^2 + 2n + 1) y^{n} = 0$$

$$-(n+1)(y^{(n+1)})_0 + (n^2 + 2n + 1)(y^{n})_0 = 0$$

$$[y^{(n+1)}]_0 = \frac{-(n^2 + 2n + 1)[y^{n}]_0}{-[n+1]}$$

$$[y^{(n+1)}]_0 = \frac{(n+1)\cancel{(n+1)}[y^{n}]_0}{\cancel{(n+1)}}$$

$$[y^{(n+1)}]_0 = (n+1)[y^{n}]_0$$

when $n = 0$

$$[y^{(0+1)}]_0 = (0+1)[y^{0}]_0$$

$$[y^{(1)}]_0 = [y^{(0)}]_0$$

when $n = 1$

$$[y^{(1+1)}]_0 = (1+1)[y^{1}]_0$$

$$[y^{(2)}]_0 = 2[y^{1}]_0$$

when $n = 2$

$$[y^{(2+1)}]_0 = (2+1)[y^{2}]_0$$

$$[y^{(3)}]_0 = 3[y^{(2)}]_0$$

$$= 3 \cdot 2 [y^{(1)}]_0$$

$$= 6 [y^{(1)}]_0$$

when $n = 3$

$$[y^{(3+1)}]_0 = (3+1)[y^{3}]_0$$

$= 4[y^{(3)}]_0$

$= 4 \cdot 6[y^{(1)}]_0$

$= 24[y^{(1)}]_0$

when $n = 4$

$[y^{(4+1)}]_0 = [4+1][y^{(4)}]_0$

$[y^{(5)}]_0 = 5(y^{(4)}]_0$

$= 5 \cdot 24[y^{(1)}]_0$

$= 120[y^{(1)}]_0$

when $= 5$

$[y^{(5+1)}]_0 = [5+1][y^{(5)}]_0$

$[y^{(6)}]_0 = 6[y^{(5)}]_0$

$= 6 \cdot 120[y^{(1)}]_0$

$= 720[y^{(1)}]_0$

when $n = 6$

$[y^{(6+1)}]_0 = [6+1][y^{6}]_0$

$[y^{(7)}_0] = 7[y^{(6)}]_0$

$= 7 \cdot 720[y^{(1)}]_0$

$= 5040[y^{(1)}]_0$

$y = (y^{0})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)}]_0 + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^{(4)})_0 + \frac{x^5}{5!}(y^{(5)})_0$

$\ldots + \frac{x^6}{6!}(y^{(6)})_0 + \frac{x^7}{7!}(y^{(7)})_0 + \cdots$

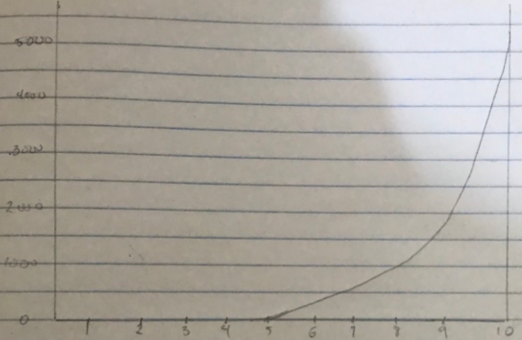
$y = (y^{0})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}[2(y^{(1)})_0] + \frac{x^3}{3!}[6(y^{(1)})_0] + \frac{x^4}{4!}[24(y^{(1)})_0] + \ldots$

$\ldots \frac{x^5}{5!}[120(y^{(1)})_0] + \frac{x^6}{6!}[720(y^{(1)})_0] + \frac{x^7}{7!}[5040(y^{(1)})_0]$

$y = [y^{0}] + x[y^{(1)}]_0 + x^2(y^{(1)})_0 + x^3(y^{(1)})_0 + x^4(y^{(1)})_0 + x^5(y^{(1)})_0 + x^6(y^{(1)})_0 +$

$\ldots x^7(y^{(1)})_0 + \ldots$

$y = (1+x)(y^{(0)}) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)[y^{(1)}]_0$



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