A

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Assignment III

- The model for the deformation (y) of a structural element is represented by the expression given in equation (1).

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Given that $y(0) = 0.0005m$ & $y'(0) = 0.0005$, applying Leibnitz Maclaurin method;

a) Obtain the power series solution of the model up to & including the term in x^7.

b) Estimate the approximate deformation when $x = 5, 8$ & $10m$.

c) With the aid of a MATLAB m-file program, plot the response of the structural element for $0 \le x \le 10m$.

Solution

a) $x(x-1)y'' + (3x-1)y' + y = 0$

$y(0) = 0.0005$ ....., $x^7 = ...$

$\Rightarrow xy''(x-1) + (3x-1)y' + y = 0$.

$= y''x(x-1) - w_1 \quad y'(3x-1) - w_2 \quad y - w_3$.

$\underline{w_1} - y''x(x-1) \Rightarrow y^{(2)} \quad x^2 - x$

$u = y^{(2)}$ $\qquad v = x^2 - x$

$u^{(n)} = y^{(n+2)}$ $\qquad v^{(1)} = 2x - 1$

$u^{(n-1)} = y^{(n+1)}$ $\qquad v^{(2)} = 2$

$u^{(n-2)} = y^{(n)}$ $\qquad v^{(3)} = 0$.

$w_2 - (3x-1)y^{(1)}$

$u = y^{(1)}$ $\quad v = 3x-1$

$u^{(n)} = y^{(n-1)}$ $\quad v' = 3$

$u^{(n-1)} = y^{(n)}$ $\quad v'' = 0$

$w_3 - y$

$u = y$ $\quad v = 1$

$u^{(n)} = y^{(n)}$ $\quad v' = 0$

Applying Leibnitz;

$$y^{(n)} = u^{(n)}v + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v^{2} + \frac{n(n-1)(n-2)}{3!}\ldots$$

$$\ldots u^{(n-3)}v^{(3)} + \frac{n(n-1)(n-2)(n-3)}{4!}u^{(n-4)}v^{(4)} + \ldots$$

Substituting;

$$w_1 = y^{(n+2)}(x^2-x) + ny^{(n+1)}(2x-1) + \frac{n(n-1)}{2!}y^{(n)}(2) + 0$$

$$w_2 = y^{(n+1)}(3x-1) + ny^{(n)}3 + 0$$

$$w_3 = y^{(n)} - 1 + 0$$

$$\Rightarrow y^{(n)} \Rightarrow y^{(n+2)}x(x-1) + ny^{(n+1)}(2x-1) + n(n-1)y^{(n)} + y^{(n+1)}\ldots$$

$$(3x-1) + ny^{(n)}(3) + y^{(n)} = 0$$

$$\Rightarrow y^{(n+2)}(x^2-x) + ny^{(n+1)}(2x-1) + n(n-1)y^{(n)} + y^{(n+1)}(3x-1) + \ldots$$

$$\ldots + 3ny^{(n)} + y^{(n)} = 0$$

$$\Rightarrow y^{(n+2)}(x^2-x) + (2xn-n)y^{(n+1)} + (n^2-n)y^{(n)} + (3x-1)y^{(n+1)} + \ldots$$

$$\ldots + 3ny^{(n)} + y^{(n)} = 0$$

$$\Rightarrow y^{(n+2)}(x^2-x) + (2xn-n)y^{(n+1)} + (3x-1)y^{(n+1)} + (n^2-n)y^{(n)} + \ldots$$

$$\ldots + 3ny^{(n)} + y^{(n)} = 0$$

$$\Rightarrow y^{(n+2)}(x^2-x) + y^{(n+1)}[2xn + 3x - n - 1] + y^{(n)}[n^2 - n + 3n + 1] = 0$$

$$\Rightarrow y^{(n+2)}(x^2-x) + y^{(n+1)}[2xn + 3x - n - 1] + y^{(n)}[n^2 + 2n + 1] = 0$$

When $x=0$

$\Rightarrow \cancel{y^{(n+2)}(0^2-0)} + y^{(n+1)}(\cancel{2(0)}n+\cancel{3(0)}-n-1) + y^{(n)}(n^2+2n+1) = 0$.

$\Rightarrow \left(y^{(n+1)}\right)_0(-n-1) + \left(y^{(n)}\right)_0(n^2+2n+1)$

$= -(n+1)\left(y^{(n+1)}\right)_0 = -(n^2+2n+1)\left(y^{(n)}\right)_0$

$$\left(y^{(n+1)}\right)_0 = \frac{+(n^2+2n+1)}{+(n+1)}\left(y^{(n)}\right)_0$$

when $n=0$:

$$y^{(0+1)} = \frac{0^2+2(0)+1}{(0+1)}y^{(0)}$$

$$y^{(1)} = 1y^{(0)}$$

$n=1$
$$y^{(1+1)} = \frac{1^2+2(1)+1}{(1+1)}y^{(1)}$$

$$y^{(2)} = 2y^{(1)}$$

$n=2$
$$y^{(2+1)} = \frac{2^2+2(2)+1}{(2+1)}y^{(2)}$$

$$= 3y^{(2)} = (3)(2)y^{(1)}$$

$$y^{(3)} = 6y^{(1)}$$

$n=3$
$$y^{(3+1)} = \frac{3^2+2(3)+1}{(3+1)}y^{(3)}$$

$$= 4y^{(3)} = (4)(6)y^{(1)}$$

$$y^{(4)} = 24y^{(1)}$$

$+1) = 0.$

$n=4$ $\quad y^{(4+1)} = \dfrac{4^2+2(4)+1}{(4+1)} y^{(4)}$

$= 5y^{(4)} = (5)(24)y^{(1)}$

$y^5 = 120y^{(1)}$

$n=5$ $\quad y^{(5+1)} = \dfrac{5^2+2(5)+1}{(5+1)} y^{(5)}$

$= 6y^{(5)} = (6)(120)y^{(1)}$

$y^{(6)} = 720y^{(1)}$

$n=6$ $\quad y^{(6+1)} = \dfrac{6^2+2(6)+1}{(6+1)} y^{(6)}$

$= 7y^{(6)} = (7)(720)y^{(1)}$

$y^{(7)} = 5040y^{(1)}$

Applying MacLaurin's theorem;

$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^{(4)})_0 + \frac{x^5}{5!}(y^{(5)}) + \dots$

$\dots + \frac{x^6}{6!}(y^{(6)})_0 + \frac{x^7}{7!}(y^{(7)})_0 + \dots$

Substituting;

$y = y^{(0)} + xy^{(0)} + \frac{x^2}{2!}2y^{(1)} + \frac{x^3}{3!}6y^{(1)} + \frac{x^4}{4!}24y^{(1)} + \dots$

$\dots + \frac{x^5}{5!}120y^{(1)} + \frac{x^6}{6!}720y^{(1)} + \frac{x^7}{7!}5040y^{(1)} + \dots$

$$y = y^{(0)}(1+x) + y^{(1)}\left[\frac{2x^2}{2!} + \frac{6x^3}{3!} + \frac{24x^4}{4!} + \frac{120x^5}{5!} + \dots \right.$$

$$\left. \dots + \frac{720x^6}{6!} + \frac{5040x^7}{7!} + \dots\right]$$

$\therefore$

$y = (y^{(0)})_0(1+x) + (y^{(1)})_0(x^2+x^3+x^4+x^5+x^6+x^7+\dots)$

$y = (1+x)(y^{(0)})_0 + (x^2+x^3+x^4+x^5+x^6+x^7+\dots)(y^{(1)})$

b.) $y_{(0)} = 0.0005\,m$, $y^{(1)}_{(0)} = 0.0005$.
deformation when $x = 5, 8$ & 10

$\therefore y = (1+x)(y^{(0)}) + (x^2+x^3+x^4+x^5+x^6+x^7)(y^{(1)})$.
$\Rightarrow y = (1+x)(0.0005\,m) + (x^2+x^3+x^4+x^5+x^6+x^7)(0.0005\,m)$

when $x = 5$.
$y = (1+5)(0.0005) + (5^2+5^3+5^4+5^5+5^6+5^7)(0.0005)$
$= 0.003 + 48.825$
$y = 48.828\,m$ //

when $x = 8$.
$y = (1+8)(0.0005) + (8^2+8^3+8^4+8^5+8^6+8^7)(0.0005)$
$= 0.0045 + 1198.368$
$y = 1198.3725\,m$ // $\simeq 1198.37\,m$ //

when $x = 10$
$y = (1+10)(0.0005) + (10^2+10^3+10^4+10^5+10^6+10^7)(0.0005)$
$= 0.0055 + 5555.55$.
$y = 5555.5555\,m$
$y \simeq 5555.56\,m$ //

c.) MATLAB
response for $0 \leq x \leq 10\,m$.

1) MATLAB.

Command window

```
Clear
clc
Close all
syms x.
x = [(1+x)*(0.0005)] + [(x^2 + x^3 + x^4 + x^5 + x^6 + x^7)*(0.0005)]
t = 0:0.01:10
xt = Subs (x,t)
xtn = double (xt)
plot (t, xtn)
x label ('t')
y label ('x')
grid on
grid minor
axis tight.
```