Applying Leibnitz method

$x(x-1)y'' + (3x-1)y' + y = 0$

for $G_1 - x(x-1)y''$

$U = y''$, $V = x^2 - x$

$U' = y''$, $V' = 2x - 1$

$U^n = y^{n+2}$ $V'' = 2$

$V''' = 0$

$U^n V + n U^{n-1} V' + \frac{n(n-1)}{2} U^{n-2} V'' + \frac{n(n-1)(n-2)}{3} U^{n-3} V'''$

$(y^{n+2})(x^2-x) + n(y^{n+1})(2x+1) + n(n-1)(y^{n})$

$x^2 - x(y^{n+2}) + 2xn - n(y^{n+1}) + n^2 - n(y^n)$

for $G_2 - (3x-1)y'$

$V = 3x-1$ $U = y'$

$V' = 3$ $U^n = y^{n+1}$

$V'' = 0$

$(y^{n+1})(3x-1) + n(y^n)(3) +$

for G_3

$U^n = y^n$, $V = 1$

y^n

Combining

$$x^2-x(y^{n+2})+n(2x-1)(y^{n+1})+n(n-1)(y^n)+(3x-1)(y^{n+1})$$
$$+3n(y^n)\cancel{\$}+y^n$$

solving

$$\Rightarrow x^2-x(y^{n+2})+(y^{n+1})(n(2x-1)+(3x-1))+(n(n-1)+3n+1)$$
$$y^n$$

$$\Rightarrow x^2-x(y^{n+2})+(n(2x-1)+3x-1)y^{n+1}+(n^2+2n+1)y^n$$

$$x^2-x(y^{n+2})+(n(2x-1)+(3x-1)]y^{n+1}+(n^2+2n+1)y^n=0$$

at $x=0$

y^n

$$(n(-1)+(-1)]y^{n+1}+(n^2+2n+1)y^n=0$$
$$(-n-1)y^{n+1}+(n^2+2n+1)y^n=0$$

$$(y^{n+1})_0=\frac{-(n^2+2n+1)y^n}{-n-1}$$

$$=\frac{\cancel{-}(n^2+2n+1)y^n}{\cancel{-}(n+1)}$$

$$(y^{n+1})_0=\frac{n^2+2n+1}{n+1}(y^n)_0$$

$$y^{n+1}=n+1\ y^n$$

at $n = 0$

$y'_{(0)} = 1 y_0$

$x = 0,\ n = \cancel{1}$

$y^{2'}_0 = 2y' = 2$

$n = 2$

$y^3 = 3y^2 = 3(2)(y') = \cancel{6}y'$

$n = 3$

$y^4 = 4y^3 = 4(3)(2)(y')_0 = \cancel{24}y'$

$n = 4$

$y^5 = 5y^4 = (2)(3)(4)(5)(y')_0$

$n = 5$

$y^6 = 6y^5 = (2)(3)(4)(5)(y')_0 = (2)(3)(4)(5)(6)$

$n = 6$

$y^7 = 7y^6 = (2)(3)(4)(5)(6)(y')_0\ (2)(3)(4)(5)(6)(7)$

Maclaurin Series.

$$y = y_0 + x(y'_0) + \frac{x^2}{2!}(y^2) + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4) + \frac{x^5}{5!}(y^5)_0 +$$

$$\frac{x^6}{6!}(y^6) + \frac{x^7}{7!}(y^7)_0$$

$$y = (y_0) + x(y'_0) + \frac{x^2}{2!}(2\,y'_0) + \frac{x^3}{3!}(3!\,y'_0) + \frac{x^4}{4!}(4!\,y'_0) +$$

$$\frac{x^5}{5!}(5!\cdot y'_0) + \frac{x^6}{6!}(6!\,y'_0) + \frac{x^7}{7!}(7!\,y'_0)$$

$$y = y_0 + y'_0 x + y' x^2 + y' x^3 + x^4 y' + x^5 y' + x^6 y' + x^7 y'$$

$$y = y_0 + y'[x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7]$$

Recall

$y_1 = y_0$

$$y = y_0[1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7]$$

(b) $y'(0) = 0.0005$

When $x = 5$

$$y_5 = 0.0005[1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7]$$

$= 0.0005[97656]$

$= 48.828 = \underline{49}$

When $x = 8$, $y_0 = 0.0005$

$$y_8 = 0.0005[1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7]$$

$= 0.0005[2396745]$

$= 1198.3725 = \underline{1198}$

$x = 10m$

$y_{10} = 0.0005[1+10+10^2+10^3+10^4+10^5+10^6+10^7]$

$= 0.0005[11111111]$

$= 5555.5555$

$\simeq \underline{\underline{5556}}$

(c) Command window

clc

clear all

close all

syms x, y

x = 0:0.1:10

y = 0.0005[1+10+10^2+10^3+10^4+10^5+10^6
+10^7]

yn = sub(y)

plot x, yn

Axis tight

Grid on

Grid minor

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