

IKETAKU .UJCU. DIVIDE

18/ETG02/047

COMPUTER ENGINEERING

Solution

$$(i) A = \int_a^b y \, dx$$

$$\int_1^2 y = 3e^{2x} \quad \& \quad y = 3e^{-x}$$

$$x = 1 \quad \& \quad x = 2$$

$$A = \int_1^2 (3e^{2x} - 3e^{-x}) \, dx$$

$$= \int_1^2 3e^{2x} \, dx - \int_1^2 3e^{-x} \, dx$$

$$= 3 \int_1^2 e^{2x} \, dx - 3 \int_1^2 e^{-x} \, dx$$

$$= 3 \left[ \frac{e^{2x}}{2} \right]_1^2 - 3 \left[ \frac{e^{-x}}{-1} \right]_1^2$$

$$= 3 \left[ \frac{e^{2(2)}}{2} - \frac{e^{2(1)}}{2} \right] - 3 \left[ -e^{-2} + e^{-1} \right]$$

$$= 3 \left[ \frac{e^4}{2} - \frac{e^2}{2} \right] - 3 \left[ -e^{-2} + e^{-1} \right]$$

$$= 3 \left[ \frac{e^4 - e^2}{2} \right] - 3 \left[ -e^{-2} + e^{-1} \right]$$

$$= 3(23.6075) - 3(0.2325)$$

$$= 70.8135 - 0.6975$$

$$A = \underline{\underline{70.116 \text{ units}^2}}$$

$$(b) A = \int_a^b y \, dx$$

$$y = 2 \sin \frac{\pi x}{10} \quad ; \quad x = 0 \quad \& \quad x = 10$$

$$A = \int_0^{10} 2 \sin \frac{\pi x}{10} \, dx$$

$$x = 2 + 2t - 2 \cos \frac{\pi x}{10}$$

$$\frac{dx}{dt} = 2 + \frac{2 \sin \pi t}{10}$$

$$dx = 2 + \frac{2 \sin \pi t}{10} dt$$

$$= \int_0^{10} \left( \frac{2 \sin \pi t}{10} \right) \left( 2 + \frac{2 \sin \pi t}{10} dt \right)$$

$$= \int_0^{10} \left( \frac{4 \sin \pi t}{10} + \frac{4 \sin^2 \pi t}{10} dt \right)$$

$$= 4 \int_0^{10} \frac{\sin \pi t}{10} + \frac{\sin^2 \pi t}{10} dt$$

$$= 4 \left[ \left( \frac{-\cos \pi t}{10} - \frac{\cos^2 \pi t}{10} \right) \right]_0^{10}$$

$$= 4 \left[ \left[ \frac{-\cos \pi}{10} (10) - \frac{\cos^2 \pi}{10} (10) \right] - \left[ \frac{\cos \pi}{10} (0) - \frac{\cos^2 \pi}{10} (0) \right] \right]$$

$$= 4 \left[ (-\cos \pi - \cos^2 \pi) - (-1 + 1) \right]$$

$$= 4 \left( -\cos \pi - \cos^2 \pi \right)$$
$$\cos^2 \pi = (\cos 2\pi + 1) \frac{1}{2}$$

$$= 4 \left( -\cos \pi - (\cos 2\pi + 1) \frac{1}{2} \right)$$

$$= 4 \left( -0.9985 - \frac{1.994}{2} \right)$$

$$= 4 \left( -0.9985 - 0.997 \right)$$

$$= 4 \left( -1.9955 \right)$$

$$= -7.982 \text{ units}^2$$