$$(y^{n+1})_0 = \frac{\cancel{t}(\cancel{n+1})(n+1)\, y^{(n)}}{\cancel{t}(\cancel{n+1})}$$

$(y^{(n+1)})_0 = (n+1)(y^{(n)})_0$ ⟶ Recurrence Equation

when $n = 0$

$(y^{1})_0 = (y^{0})_0$, since $(y^{0})_0 = (y^{(0)})_0 = 0.0005 = 0.0005$

when $n = 1$

$(y^{2})_0 = 2(y^{1})_0 = 2(y^{0})_0$

when $n = 2$

$(y^{(3)})_0 = 3(y^{(2)})_0 = 3 \times 2(y^{0})_0 = 3!(y^{0})_0$

when $n = 3$

$(y^{(4)})_0 = 4(y^{(3)})_0 = 4 \times 3 \times 2(y^{0})_0 = 4!(y^{(0)})$

when $n = 4$

$(y^{(5)})_0 = 5(y^{(4)})_0 = 5 \times 4 \times 3 \times 2(y^{0})_0 = 5!(y^{(0)})_0$

when $n = 5$

$(y^{(6)})_0 = 6(y^{(5)})_0 = 6 \times 5 \times 4 \times 3 \times 2(y^{0})_0 = 6!(y^{(0)})_0$

when $n = 6$

$(y^{(7)})_0 = 7(y^{(6)})_0 = 7 \times 6 \times 5 \times 4 \times 3 \times 2(y^{0})_0 = 7!(y^{0})_0$

Maclaurin

$$y = (y^{0})_0 + x(y^{0})_0 + \frac{x^2}{\cancel{2!}} \cdot \cancel{2}(y^{0})_0 + \frac{x^3}{\cancel{3!}} \cdot \cancel{3!}(y^{(0)})_0 + \frac{x^4}{\cancel{4!}} \cdot \cancel{4!}(y^{(0)})_0 + \frac{x^5}{\cancel{5!}} \cdot \cancel{5!}(y^{0})_0 + \frac{x^6}{\cancel{6!}} \cdot \cancel{6!}(y^{(0)})_0 + \frac{x^7}{\cancel{7!}} \cdot \cancel{7!}(y^{(0)})_0$$

$$y = (y^{(0)})_0 + x(y^{(0)})_0 + x^2(y^{(0)})_0 + x^3(y^{(0)})_0 + x^4(y^{(0)})_0 + x^5(y^{(0)})_0 + x^6(y^{(0)})_0 + x^7(y^{(0)})_0$$

$$y = (y^{(0)})_0(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

b) when $x = 5m$, $(y^{0})_0 = 0.0005m$

$y = 0.0005(1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)$

$= 0.0005(97656)$

$y = 48.828m$

when $x = 8m$

$y = 0.0005(1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$

$y = 0.0005(2396745)$

$y = 1198.3725\,m$

when $x = 10\,m$

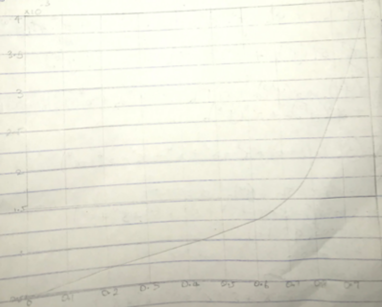
$y = 0.0005(1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$

$= 0.0005(11111111)$

$y = 5555.5555\,m$

c) Commandwindow

```
clear
clc
close all
syms x
x = 0:0.01:1
y=0.0005*(1 + x + x.^2 + x.^3 + x.^4 + x.^5 + x.^6 + x.^7)
plot(x,y)
grid on
grid minor
```



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ENG 381 Assignment 3

Equation (1);

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Given that $y(0) = 0.0005m$ and $y'(0) = 0.0005$, applying Leibnitz-Maclaurin Method;

a) Obtain the power series solution of the model up to and including the term in x^7

solution

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Leibnitz

$x(x-1)y''$

$V = x^2 - x$ $\quad u = y''$

$V' = 2x - 1$ $\quad u^{(n)} = y^{(n+2)}$

$V^{(2)} = 2$ $\quad u^{(n-1)} = y^{(n+1)}$

$u^{(n-2)} = y^{(n)}$

$(3x-1)y'$

$V = 3x-1$ $\quad u = y'$

$V^{(1)} = 3$ $\quad u^{n} = y^{(n+1)}$

$u^{(n-1)} = y^{(n)}$

$\frac{y}{y^{(n)}}$

Adding

$$x^2-x(y^{(n+2)}) + n\cdot(2x-1)y^{(n+1)} + \frac{n(n-1)}{\cancel{2}}\cdot\cancel{2}\cdot y^{(n)} + (3x-1)\cdot y^{(n+1)} +$$

$$n\cdot 3\cdot y^{(n)} + y^{(n)} = 0$$

$$x^2-x(y^{(n+2)}) + (2x-1)ny^{(n+1)} + n^2-n\cdot y^{(n)} + (3x-1)y^{(n+1)} + 3ny^{(n)}$$

$$+ y^{(n)} = 0$$

$$x^2-x(y^{(n+2)}) + (2xn-n+3x-1)y^{(n+1)} + (n^2-n+3n+1)y^{(n)} = 0$$

$$x^2-x(y^{(n+2)}) + (2xn-n+3x-1)y^{(n+1)} + (n^2+2n+1)y^{(n)} = 0$$

at $x = 0$

$$0^2 - 0(y^{(n+2)}) + (2\times 0\times n - n + (3\times 0) - 1)y^{(n+1)} + (n^2+2n+1)y^{(n)} = 0$$

$$(-n-1)y^{(n+1)} + (n^2+2n+1)y^{(n)} = 0$$

$$-(n+1)y^{(n+1)} + (n^2+2n+1)y^{(n)} = 0$$

$$-(n+1)y^{(n+1)} = -(n^2+2n+1)y^{(n)}$$

$$(y^{n+1})_0 = \frac{-(n^2+2n+1)y^{(n)}}{-(n+1)}$$