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MECH ENGR

(a) Solution

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Part a

$x(x-1)y''$, $u = y'$, $u^n = y^{n+2}$

$v = x^2 - x$, $v' = 2x - 1$, $v'' = 2$

$$y = u^n v + n u^{n-1} v' + \frac{n(n-1)y^{n-2}v''}{2!}$$

$$= x(x-1)y^{n+2} + n y^{n+1}(2x-1) + \frac{n(n-1)y^n 2}{2}$$

$$= x(x-1)y^{n+2} + n y^{n+1}(2x-1) + (n^2 - n)y^n$$

Part b

$(3x-1)y'$, $u = y'$, $y^n = y^{n+1}$

$v = 3x - 1$, $v' = 3$

$y^n = u^n v + n u^{n-1} v' +$ ~~$n(n-1)x^{n-2}$~~

$= (3x-1)y^{n+1} + n(3)y^n$

Part c

y, $u = y$, $y^n = y^n$

$\therefore$ part a + part b + part c $= 0$

$$x(x-1)y^{n+2} + n y^{n+1}(2x-1) + (n^2 - n)y^n + (3x-1)y^{n+1} + 3n y^n + y^n$$

$\therefore$

$$x(x-1)y^{n+2} + (2xn - n + 3x - 1)y^{n+1} + (n^2 + 2n + 1)y^n = 0$$

when $x = 0$

$$(2(0)n - n + (3(0)) - 1)y^{n+1} + (n^2 + 2n + 1)y^n = 0$$

$$-(n+1)y^{n+1} = -[(n+1)(n+1)]y^n$$

$$y^{n+1} = (n+1)y^n$$

wodu. $y^{n+1} = (n+1)y^n$

when $n = 0$

$y_0^1 = y_0$

when $n = 1$

$y_0^2 = 2y_0^1$, $2(y_0)$

when $n = 2$

$y_0^3 = 3y_0^{II}$, $3(2y_0) = 6y_0$

when $n = 3$

$y_0^4 = 4y_0^{III}$ $4(6y_0) = 24y_0$

when $n = 4$

$y_0^5 = 5y_0^{IV}$ $5(24y_0) = 120y_0$

when $n = 5$

$y_0^6 = 6y_0^V$ $6(120y_0) = 720y_0$

when $n = 6$

$y_0^7 = 7y_0^{VI}$ $7(720y_0) = 5040y_0$

when $n = 7$

~~$y_0^8 = 8y_0^{VII}$ $8(5040y_0) = 40320y_0$~~

from Mclaurin Series

$$y = \frac{x^0 y_0}{0!} + \frac{x^1 y_0^1}{1!} + \frac{x^2 y_0^{II}}{2!} + \frac{x^3}{3!} y_0^{III} + \frac{x^4 y_0^{IV}}{4!} + \frac{x^5 y_0^V}{5!} + \frac{x^6 y_0^{VI}}{6!}$$

$$+ \frac{x^7 y_0^{VII}}{7!} + \cancel{\frac{x^8 y_0^{VIII}}{8!}}$$

$$y = y_0 + xy_0 + \frac{x^2 2y_0}{2!} + \frac{x^3 6y_0}{3!} + \frac{x^4 24y_0}{4!} + \frac{x^5 120y_0}{5!} + \frac{x^6 720y_0}{6!}$$

$$+ x^7 \frac{5040 y_0}{7!} + \cancel{x^8 \frac{40320 y_0}{8!}}$$

$$y = y_0 + xy_0 + x^2 y_0 + x^3 y_0 + x^4 y_0 + x^5 y_0 + x^6 y_0 + x^7 y_0 + \dots$$

$$y = y_0 \left[1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \dots \right]$$

where $y_0 = 0.0005$ & $y'_0 = 0.0005$
where $y_0 = y'_0$

5) When $x = 5$

$y = 0.0005 [1 + 5 + 25 + 125 + 625 + 3125 + 15625 + 78125$
$+ 390625]$

~~$y = 0.0005 [488.281]$~~

~~$y = 244.14$~~ $y = 0.0005 [976562]$

$y = 48.828$

when $x = 8$

$y = 0.0005 [1 + 8 + 64 + 512 + 4096 + 32768 + 262144 + 2,097,152]$

$y = 0.0005 [2,396,745]$

$y = 1198.3725$

when $x = 10$

$y = 0.0005 [1 + 10 + 100 + 1000 + 10000 + 100000 + 1000000 + 10000000 + 100000000]$

$y = [1,111,111] 0.0005$

$y = 5,555.555$

c) Commandwindow

```
clear
clc
Syms x
Syms y
x = (0:10);
y = 0.0005 * (1 + x + (x.^2) + (x.^3) + (x.^4)
    + (x.^5) + (x.^6) + (x.^7));
plot (x, y)
grid on
grid minor
```