at $n=5$

$(y^5)_0 = (5+1)(y^5)_0$

$= 6(y^5)_0$

$= 6 \times 120 (y')_0$

$(y^6)_0 = 720(y')_0$

at $n=6$

$(y^7)_0 = (6+1)(y^6)_0$

$= 7(y^6)_0$

$= 7 \times 720 (y')_0$

$(y^7)_0 = 5040(y')_0$

from maclawin series.

$y = (y)_0 + x(y')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \cdots$

$y = (y)_0 + x(y')_0 + \frac{x^2}{2!}(2(y')_0) + \frac{x^3}{3!}(6(y')_0) +$

$\frac{x^4}{4!}(24(y')_0) + \frac{x^5}{5!}(120(y')_0) + \frac{x^6}{6!}(720(y')_0)$

$+ \frac{x^7}{7!}(5040(y')_0) + \cdots$

$y = (y)_0 + x(y')_0 + x^2(y')_0 + x^3(y')_0 + x^4(y')_0 + x^5(y')_0 +$

$x^6(y')_0 + x^7(y')_0 + \cdots$

$y = (y)_0 + (y')_0 [x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 \cdots]$

but $(y)_0 = 0.0005m$ and $(y')_0 = 0.0005$

$\therefore y = 0.0005 + 0.0005 [x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7]_4$

b) when x=5m,

$y = 0.0005 + 0.0005[5+25+125+625+3125+15625+78125]$

$y = 0.0005 + 0.0005[97655]$

$y = 48.8275m$

when x=8m

$y = 0.0005 + 0.0005[8+64+512+4096+32768+262144+2097152]$

$y = 0.0005 + 0.0005[2396744]$

$y = 1198.3725m$

when x=10m

$y = 0.0005 + 0.0005[10+100+1000+10000+100000+1000000+10000000]$

d) $y = 0.0005 + 0.0005[11111110]$

$y = 5555.5555$

c) command window

```
clear
clc
syms x
syms y
x = (0:10)
y = 0.0005 + 0.0005*[(x+(x^2)+(x^3)+(x^4)+(x^5)+(x^6)+(x^7)]
plot(x,y)
grid on
grid minor
xlabel('x')
ylabel('Structural deformation')
```

Mechanical engineering
Bakareeye ogheneriega paul
ulengo6/074
Solution

a) $x(x-1)y'' + (3x-1)y' + y = 0$

for the nth derivative

$x(x-1)y^{n+2} + (2x-1)ny^{n+1} + n(n-1)y^n + (3x-1)y^{n+1} + 3ny^n + y^n = 0$

$x(x-1)y^{n+2} + y^{n+1}(2xn - n + 3x - 1) + y^n(n^2 - n + 3n + 1) = 0$

at $x = 0$

$(y^{n+1})_0(-n-1) + (y^n)_0(n^2+2n+1) = 0$

$-(n+1)(y^{n+1})_0 = -(n^2+2n+1)(y^n)_0$

$(n+1)(y^{n+1})_0 = (n^2+2n+1)(y^n)_0$

$(y^{n+1})_0 = (n+1)(y^n)_0$

at $n = 0$

$(y^{0+1})_0 = (0+1)(y^0)_0$

$(y')_0 = (y)_0$

at $n = 1$

$(y^{1+1})_0 = (1+1)(y')_0$

$(y^2)_0 = 2(y')_0$

at $n = 2$

$(y^{2+1})_0 = (2+1)(y^2)_0$

$(y^3)_0 = 3(y^2)_0$

$(y^3)_0 = 6(y')_0$

at $n = 3$

$(y^4)_0 = (3+1)(y^3)_0$

$(y^4)_0 = 4(y^3)_0 = 4\times6(y')_0 = 24(y')_0$

at $n = 4$

$(y^5)_0 = (4+1)(y^4)_0$

$= 5(y^4)_0$

$= 5\times24(y')_0$

$(y^5)_0 = 120(y')_0$