Assignment ~~Assignment~~

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17/Engo4/050
Elect/Elect

(1) $x(x-1)y'' + (3x-1)y' + y = 0$

Taking $x(x-1)y'' = w_1$

$(3x-1)y' = w_2$

$y = w_3$

Considering $w_1$

$U = y''$ $\quad V = x(x-1)$

$U^n = y^{(n+2)}$ $\quad V' = 2x-1$

$U^{n-1} = y^{(n+1)}$ $\quad V'' = 2$

$U^{n-2} = y^2$ $\quad V''' = 0$

$w_2$

$U = y'$ $\quad V = (3x-1)$

$U^n = y^{(n+1)}$ $\quad V' = 3$

$U^{n-1} = y^n$ $\quad V'' = 0$

$w_3$

$U = y$ $\quad V = 1$

$U^n = y^n$ $\quad V' = 0$

$$y^n = U^n V + nU^{(n-1)}V' + \frac{n(n-1)}{2!}U^{(n-2)}V'' + \frac{n(n-1)(n-2)}{3!}U^{n-3}V''' + \cdots$$

$w_1^n = y^{n+2}.(x^2-x) + ny^{n+1}.(2x-1) + \frac{n(n-1)}{2!}y^n.2 + 0$

$w_2^n = y^{(n+1)}.(3x-1) + ny^n.3 + 0$

$w_3^n = y^n.1 + 0$

$w_1 + w_2 + w_3$

$y^{n+2}.(x^2-x) + ny^{n+1}.(2x-1) + \frac{n(n-1)}{2}y^n.2 + y^{(n+1)}.(3x-1) +$

$ny^n.3 + y^n = 0$

$(x^2-x)y^{(n+2)} + (2x-1)ny^{(n+1)} + (3x-1)y^{n+1} + n(n-1)y^n + 3ny^n + y^n = 0$

$(x^2-x)y^{(n+2)} + (2xn-n)y^{(n+1)} + (3x-1)y^{(n+1)} + (n^2-n)y^n + 3ny^n + y^n = 0$

$(x^2-x)y^{(n+2)} + (2xn+3x-n-1)y^{(n+1)} + (n^2+n+3n+1)y^n = 0$

$(x^2-x)y^{(n+2)} + (2xn+3x-n-1)y^{n+1} + (n^2+2n+1)y^n = 0$

$(x-1)xy^{n+2} + (2xn+3x-n-1)y^{n+1} + (n^2+2n+1)y^n = 0$

When $x=0$

$(0-1)0y^{(n+2)} + (2(0)n+3(0)-n-1)y^{n+1} + (n^2+2n+1)y^n = 0$

$(-n-1)y^{n+1} + (n^2+2n+1)y^n = 0$

~~$= (n+1)(y$~~

$-(n+1)(y^{n+1})_0 + (n^2+2n+1)(y^n)_0 = 0$

$(y^{(n+1)})_0 = \frac{-(n^2+2n+1)(y^n)_0}{-(n+1)}$

$(y^{(n+1)})_0 = \frac{(n+1)(n+1)(y^n)_0}{1}$

$(y^{n+1})_0 = (n+1)(y^n)_0$

When $n=0$

$(y^{0+1})_0 = (0+1)(y^0)_0$

$(y^1)_0 = 1(y^{(0)})_0$

When $n=1$

$(y^{1+1})_0 = (1+1)(y^{(1)})_0$

When $n=2$

$[y^{(3)}]_0 = [2+1][y^{(2)}]_0$

$[y^{(3)}]_0 = 3[y^{(3)}]_0 = 3(2)[y^{(1)}]_0$

$[y^{(3)}]_0 = 6[y^{(1)}]_0$

When $n=3$

$(y^{(4)})_0 = (3+1)[y^{(3)}]_0$

~~$[y^{(4)}]_0 = [y^{(4)}]_0$~~

$[y^{(4)}] = 4(y^{(3)}]_0 = 4[6][y^{(1)}]_0$

$(y^{(4)})_0 = 24[y^{(1)}]_0$

When $n=4$

$[y^{(5)}]_0 = (4+1)[y^{(4)}]_0$

$[y^{(5)}]_0 = 5[y^{(4)}]_0 = 5(24)[y^{(1)}]_0$

$(y^{(5)})_0 = 120[y^{(1)}]_0$

When $n=5$

$(y^{(6)}]_0 = [5+1][y^{(5)}]_0$

$(y^{(6)}]_0 = 6[y^{(5)}]_0 = 6[120](y^{(1)}]_0$

$(y^{6})_0 = 720[y^{(1)}]_0$

When $n=6$

$(y^{(7)}]_0 = (6+1)[y^{(1)}]_0$

$= 7[y^{(6)}]_0 = 7(720)(y^{(1)})_0$

$[y^{(7)}]_0 = 5040(y^{(7)}]_0$

(a) $y = [y^{(0)}]_0 + x[y^{(1)}]_0 + \frac{x^2}{2!}(y^{(2)}]_0 + \frac{x^3}{3!}\cdot[y^{(3)}]_0 + \frac{x^4}{4!}(y^4$

$+ \frac{x^5}{5!}(y^5)_0 + \frac{x^6}{6!}(y^6]_0 + \frac{x^7}{7!}(y^{(7)}]_0$

$$y = (y^{(0)})_0 + x[y^{(1)}]_0 + \frac{x^2}{2!}\cdot 2[y^{(2)}]_0 + \frac{x^3}{3!}\cdot 6[y^{(3)}]_0 + \frac{x^4}{4!}\cdot 24(y^{(4)})_0 + \frac{x^5}{5!}\cdot 120(y^{(5)})_0 + \frac{x^6}{6!}\cdot 720[y^{(6)}]_0 + \frac{x^7}{7!}\cdot 5040(y^{(7)})_0$$

(a) $y = (1+x)[y^{(0)}]_0 + (x^2+x^3+x^4+x^5+x^6+x^7)[y^{(1)}]_0$

$y_{(0)} = 0.0005m$, $y'_{(0)} = 0.0005$

$y = (1+x)(0.0005m) + (x^2+x^3+x^4+x^5+x^6+x^7)$

$y = (1+x)(0.0005m) + (x^2+x^3+x^4+x^5+x^6+x^7)(0.0005)$

When $x = 5m$, $8m$ and $10m$

$y = (1+5)(0.0005m) + (5^2+5^3+5^4+5^5+5^6+5^7)(0.0005)$

$y = 3\times10^{-3}m + 97650m\,(0.0005)$

$y = 3\times10^{-3}m + 48.825m$

$y = 48.828m$

When $x = 8m$

$y = (1+8)(0.0005) + (8^2+8^3+8^4+8^5+8^6+8^7)(0.0005)$

$y = 4.5\times10^{-3} + 2396736\,(0.0005)$

$y = 4.5\times10^{-3} + 1198$

$y = 1198m$

When $x = 10$

$y = (1+10)(0.0005) + (10^2+10^3+10^4+10^5+10^6+10^7)(0.0005)$

$y = 5.5\times10^{-3} + 11111100\,(0.0005)$

$y = 5555.5555m$

$y = 5556m$

Matlab.

1. Command Window
2. clear
3. clc
4. Close all
5. Syms x
6. x = ((1+x)*(0.0005)+((x^2+x^3+x^4+x^5+x^6+x^7)*
7. (0.0005))
8. t = 0:0.01:10
9. xt = subs(x,t)
10. xtn = double(xt)
11. plot(t,xtn)
12. xlabel('t')
13. ylabel('x')
14. grid on
15. grid minor
16. axis tight