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Mechanical Engineering

1a) $x(x-1)y'' + (3x-1)y' + y = 0$

Part a

$x(x-1)y''$, $u = y''$, $u^n = y^{n+2}$

$v = x^2 - x$, $v = 2x-1$, $v'' = 2$

$y = u^n v + nu^{n-1}v' + \frac{n(n-1)u^{n-2}v''}{2!}$

$= x(x-1)y^{n+2} + ny^{n+1}(2x-1) + \frac{n(n-1)y^n 2}{2}$

$= x(x-1)y^{n+2} + ny^{n+1}(2x-1) + (n^2-n)y^n$

Part b

$(3x-1)y'$, $u = y'$, $u^n = y^{n+1}$

$v = 3x-1$, $v' = 3$

$y^n = u^n v + ny^{n-1}v' + n''$

$= (3x-1)y^{n+1} + 3ny^n$

Part c

y, $u = y$, $u^n = y^n$

∴ Part A + Part B + Part C = 0

$x(x-1)y^{n+2} + ny^{n+1}(2x+1) + (n^2-n)y^n + (3x-1)y^{n+1} + 3ny^n + y^n$

∴

$x(x-1)y^{n+2} + (2xn - n + 3x - 1)y^{n+1} + (n^2+2n+1)y^n = 0$

when $x = 0$

$(2(0)n - n + 3(1) - 1)y^{n+1} + (n^2+2n+1)y^n = 0$

$(n+1)y^{n+1} = (n+1)(n+1)y^n = 0$

$\therefore y^{n+1} = (n+1)y^n$

$y^{n+1} = (n+1)y^n$

when $n=0$

$y_0^1 = y_0$

when $n=1$

$y_0^2 = 2y_0^1$, $2(y_0)$

when $n=2$

$y_0^3 = 3y_0^{11}$, $3(2y_0) = 6y_0$

when $n=3$

$y_0^4 = 4y_0^{11}$ $4(6y_0) = 24y_0$

when $n=4$

$y_0^5 = 5y_0^{IV}$ $5(24y_0) = 120y_0$

when $n=5$

$y_0^6 = 6y_0^{V}$ $6(120y_0) = 720y_0$

when $n=6$

$y_0^7 = 7y^{VI}$ $7(720y_0) = 5040y_0$

From Mclauren Series

$$y = \frac{x^0 y_0}{0!} + \frac{x^1 y_0^1}{1!} + \frac{x^2 y_0^{11}}{2!} + \frac{x^3 y_0^{111}}{3!} + \frac{x^4 y_0^{IV}}{4!} + \frac{x^5 y_0^{V}}{5!} + \frac{x^6 y^{VI}}{6!} + \frac{x^7 y^{VII}}{7!}$$

$$y = y_0 + xy_0 + \frac{x^2 2y_0}{2!} + \frac{x^3 6y_0}{3!} + \frac{x^4 24y}{4!} + \frac{x^5 \cdot 120y_0}{5!} + \frac{x^6 720y_0}{6!} + \frac{x^7 5040y_0}{7!}$$

$$y = y_0 + xy_0 + x^2 y_0 + x^3 y_0 + x^4 y_0 + x^5 y_0 + x^6 y_0 + x^7 y_0.$$

$$y = y_0 [1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x]$$

where $y_0 = 0.0005$ & $y_0^1 = 0.0005$

where $y_0 = y_0^1$

b) when $x = 5m$

$y = 0.0005 [1 + 5 + 25 + 125 + 625 + 3125 + 15625 + 78125]$

$y = 0.0005\ [97656]$

$y = 48.825$

when $x = 8$

$y = 0.0005\ [1+8+64+512+4096+32768+262144+2,097,152]$

$y = 0.0005\ [2,396,745]$

$y = 1198.3725$

when $x = 10$

$y = 0.0005\ [1+10+100+1000+10000+1000000+10000000+100000000]$

$y = [11,111,111]\ 0.0005$

$y = 5,555.555$

c) Command window

```
clear
clc
Syms x
Syms y
x = (0.10)
y = 0.0005(1+x+(x.^2)+(x.*3)+(x.*4)+(x.^5)+(x.^6)+(x.^7));
plot (x,y)
grid on
grid minor
xlabel ('x')
ylabel ('Structural Deformation')
```