

Applying Leibniz method

$$x(x-1)y'' + (3x-1)y' + y = 0$$

For $G_1 = x(x-1)y''$

$$u = y'' \quad , \quad v = x^2 - x$$

$$u' = y''' \quad , \quad v' = 2x - 1$$

$$u'' = y^{(4)} \quad , \quad v'' = 2$$

$$0 = u''v + n u''v' + \frac{n(n-1)}{2} u''v'' + \frac{n(n-1)(n-2)}{3} u''v'''$$

$$(y^{(4)})(x^2-x) + n(y^{(4)})(2x-1) + n(n-1)(y^{(4)})x^2 - x(y^{(4)}) + 2xn - n(y^{(4)}) + n^2 - n(y^{(4)})$$

For $G_2 = (3x-1)y'$

$$u = y' \quad , \quad v = 3x - 1$$

$$u' = y'' \quad , \quad v' = 3$$

$$(y'')(3x-1) + n(y'')(3) +$$

for $G_3 = y^n$

$$u = y^n \quad , \quad v = 1$$

$$u' = ny^{n-1}$$

Combining

$$x^2 - x(y^{n+2}) + n(2x-1)(y^{n+1}) + n(n-1)(y^n) + (3x-1)(y^{n+1}) + 3n(y^n)$$

Solving

$$\Rightarrow x^2 - x(y^{n+2}) + (y^{n+1})(n(2x-1) + (3x-1)) + (n(n-1) + 3n + 1)y^n$$

$$\Rightarrow x^2 - x(y^{n+2}) + (n(2x-1) + 3x-1)y^{n+1} + (n^2 + 2n + 1)y^n$$

$$(x^2 - x(y^{n+2}) + (n(2x-1) + 3x-1)y^{n+1} + (n^2 + 2n + 1)y^n) = 0$$

$$\text{at } x=0, (1+2x)^{n+1} + (x-3)^{n+1}$$

$$(n-1) + (n-1)] y^{n+1} + (n^2 + 2n + 1)y^n = 0$$

$$(-n-1)y^{n+1} + (n^2 + 2n + 1)y^n = 0$$

$$\frac{(n^2 + 2n + 1)y^n}{(n+1)}$$

$$y^{n+1} = \frac{n+1}{n+1} y^n$$

$$+ \binom{n}{1} x^1 n e^0 (y')^1_0 + \binom{n}{2} x^2 + \binom{n}{3} x^3 + \dots$$

$$y^1_0 = 2y'_0 = 2$$

$$y^3_0 = 3(2y'_0)^2 = 12$$

$$y^4_0 = 4(3)(2)(y'_0) = 24$$

$$y^5_0 = 5(4)(3)(2)(y'_0) = 48$$

$$y^6_0 = 6(5)(4)(3)(2)(y'_0) = 72$$

$$y^7_0 = 7(6)(5)(4)(3)(2)(y'_0) = 1008$$

$$[1 + 2 + 12 + 24 + 48 + 72 + 1008] e^0 = 1164$$

Maclaurin Series

$$y = y_0 + x(y'_0) + \frac{x^2}{2!}(y''_0) + \frac{x^3}{3!}(y'''_0) + \frac{x^4}{4!}(y^{(4)}_0) + \frac{x^5}{5!}(y^{(5)}_0) + \dots$$

$$\frac{x^6}{6!}(y^{(6)}_0) + \frac{x^7}{7!}(y^{(7)}_0)$$

$$[1 + 2 + 12 + 24 + 48 + 72 + 1008] e^0 = 1164$$

(continued)

$$y(1) = 2018.8115$$

$$y = (y_0) + x(y'_0) + \frac{x^2}{2!} (2y''_0) + \frac{x^3}{3!} (3!y'''_0) + \frac{x^4}{4!} (4!y^{(4)}_0) + \frac{x^5}{5!} (5!y^{(5)}_0) + \frac{x^6}{6!} (6!y^{(6)}_0) + \frac{x^7}{7!} (7!y^{(7)}_0)$$

$$y = y_0 + y'_0 x + y''_0 x^2 + y'''_0 x^3 + y^{(4)}_0 x^4 + y^{(5)}_0 x^5 + y^{(6)}_0 x^6 + y^{(7)}_0 x^7$$

$$y = y_0 + y'_0 [x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7]$$

Recall

$$y_1 = y_0$$

$$y = y_0 [1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7]$$

(b) $y'_0 = 0.0005$

when $x = 5$

$$y_5 = 0.0005 [1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7]$$

$$= 0.0005 [97656]$$

$$= 48.828 \approx 49$$

when $x = 6$, $y_0 = 0.0005$

$$y_6 = 0.0005 [1 + 6 + 6^2 + 6^3 + 6^4 + 6^5 + 6^6 + 6^7]$$

$$= 0.0005 [2396745]$$

$$= 1198.3725 \approx 1198$$

$$x = 10^m$$

$$y_{10} = 0.0005 [1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7]$$

$$= 0.0005 [11111111]$$

$$= 5555.5555$$

$$\approx \underline{\underline{5556}}$$

© Command window

clc

clear all

close all

syms x, y

$$x = 0:0.1:10$$

$$y = 0.0005 [1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7]$$

$y_n = \text{Sub}(y)$

plot x, y_n

axis tight

grid on

grid minor