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Questions

1) Find the area bounded by the curves $y = 3e^{2x}$ and $y = 3e^{-x}$ and the ordinates at $x = 1$ and $x = 2$.
2) The parametric equations of a curve are $y = 2\sin \pi/10 t$ and $x = 2 + 2t - 2\cos \pi/10 t$. Find the area under the curve between $t = 0$ and $t = 10$.

Solution

1 $y = 3e^{2x}$, $y = 3e^{-x}$

$$\int_a^b y_2\,dx = \int_a^b y_1\,dx$$

$$\int_1^2 3e^{2x} - \int_1^2 3e^{-x}\,dx$$

$$\left[\frac{3e^{2(2)}}{2} - \frac{3e^{-(2)}}{-1}\right] - \left[\frac{3e^{2(1)}}{2} - \frac{3e^{-1(-1)}}{-1}\right]$$

$$= \left[\frac{3e^{2(2)}}{2} + \frac{3e^{-(2)}}{1}\right] - \left[\frac{3e^{2(1)}}{2} + \frac{3e^{-(1)}}{1}\right]$$

$$= 82.30 - 12.18$$

$$= 70.125 \text{ Sq. units.}$$

2 $y = 2\sin \pi t$ $x = 2 + 2t - 2\cos \frac{\pi}{10} t$

$$A = \int_a^b y\,dx \qquad dx = 2 + \frac{2\pi}{10}\sin \frac{\pi}{10} t$$

$$A = \int_0^{10} 2\sin \frac{\pi}{10} t\,dx$$

$$A = \int_0^{10} \left[2\sin \frac{\pi}{10} t\right] \times \left[2 \times \frac{1}{10}\pi \sin \frac{\pi}{10} t\right] dt$$

$$A = \int_0^{10} 4\sin \frac{\pi}{10} + \frac{2\pi}{5}\sin^2 \frac{\pi}{10} t\,dt$$

Recall that $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ i.e

$$\sin^2 \frac{\pi}{10} = \frac{1}{2}\left(1 - \cos \frac{2\pi}{10}\right)$$

$$A = \int_0^{10} 4\sin \frac{\pi}{10} t + 2\frac{\pi}{5} \times \frac{1}{2}\left[1 - \cos \frac{2\pi}{10} t\right] dt$$

$$A = \int_0^{10} 4\sin \frac{\pi t}{10} + \frac{2}{10}\pi\left(1 - \cos \frac{2}{10}\pi\right] t\,dt$$

$$A = \int_0^{10} 4\sin \frac{\pi t}{10} + \frac{1}{5}\pi\left[1 - \cos \frac{\pi}{10}\right] t\,dt$$

$$A = 4\int_0^{10} \frac{10}{\pi}\cos \frac{\pi}{10} t\Big]_0^{10} + \frac{\pi}{5}\left[t - \frac{10}{\pi}\sin \frac{\pi}{10} t\right]_0^{10}$$

$$A = 4\left[\frac{100}{\pi}\cos \frac{\pi t}{10}\right]_0^{10} + \pi/5\left[(10 + \frac{10}{\pi}\sin \frac{\pi}{10}(10)\right] - \left[4x - \frac{10}{\pi}\right.$$

$$\cos \frac{\pi}{10}(10) + \frac{\pi}{5}(10) - \frac{10}{\pi}\sin \frac{\pi}{10} + 0\Big]$$

$$A = [12.73 + 6.28] - [12.73 + 0]$$

$$A = 31.743 \text{ q. units}$$