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$$x(x-1)y'' + (3x-1)y' + y = 0$$

Applying Leibnitz. Method

for $G_1 \Rightarrow x(x-1)y''$

$u = y''$ $\quad v = x^2 - x$

$u' = y''$ $\quad v' = 2x - 1$

$u^n = y^{n+2}$ $\quad v'' = 2$

$v''' = 0$

$$u^n v + n u^{n-1} v' + \frac{n(n-1)}{2} u^{n-2} v + \frac{n(n-1)(n-2)}{3} u^{n-3}$$

$$(y^{n+2})(x^2 - x) + n(y^{n+1})(2x-1) + n(n-1)(y^n)$$

$$x^2 - x(y^{n+2}) + 2xn - n(y^{n+1}) + n^2 - n(y^n)$$

for $G_2 \Rightarrow (3x-1)y'$

$u = y'$ $\quad v = (3x-1)$

$u^n = y^{n+1}$ $\quad v' = 3$

$v'' = 0$

$$(y^{n+1})(3x-1) + n(y^n)(3)$$

for $G_3 \Rightarrow y$

$u^n = y^n$ $\quad v = 1$

$u = y$ $\quad v' = 0$

Combining

$$x^2 - x(y^{n+2}) + n(2x-1)(y^{n+1}) + n(n-1)(y^n) + (3x-1)(y^{n+1}) + 3n(y^n) + y^n$$

Solving

$$x^2 - x(y^{n+2}) + n(2x-1)(3x-1)(y^{n+1}) + [n(n-1) + 3n + 1]y^n$$

$$x^2 - x(y^{n+2}) + n(2x-1)(3x-1)y^{n+1} + (n^2 + 2n + 1)y^n$$

If $x = 0$

$$[n(-1) + (-1)]y^{n+1} + (n^2 + 2n + 1)y^n = 0$$

$$(-n-1)y^{n+1} + (n^2 + 2n + 1)y^n = 0$$

$$-(n+1)y^{n+1} = -(n^2 + 2n + 1)y^n$$

$$(y^{n+1})_0 = \frac{f(n+1)(n+1)y^n}{f(n+1)}$$

$$(y^{n+1})_0 = (n+1)y^n \text{ - Recurrent Equation}$$

$n = 0$

$$y^1 = y_0$$

$n = 1$

$$y^2 = 2y^1$$

$n = 2$

$$y^3 = 3y^2 = (2)(3)(y^1) = 3!(y^1)_0$$

$n = 3$

$$y^4 = 4y^3 = (2)(3)(4)(y^1) = 4!(y^1)_0$$

$n = 4$

$$y^5 = 5y^4 = (2)(3)(4)(5)(y^1) = 5!(y^1)_0$$

$n = 5$

$$y^6 = 6y^5 = (2)(3)(4)(5)(6)(y^1) = 6!(y^1)_0$$

$n = 6$

$$y^7 = 7y^6 = (2)(3)(4)(5)(6)(7)(y^1) = 7!(y^1)_0$$

Maclaurin Series

$$y = y_0 + x(y^1)_0 + \frac{x^2}{2!}(y^2) + \frac{x^3}{3!}(y^3) + \frac{x^4}{4!}(y^4) + \frac{x^5}{5!}(y^5) + \frac{x^6}{6!}(y^6) + \frac{x^7}{7!}(y^7)$$

$$y = y_0 + x(y^1)_0 + \frac{x^2}{2!}(2y^1_0) + \frac{x^3}{3!}(3!y^1_0) + \frac{x^4}{4!}(4!y^1_0) + \frac{x^5}{5!}(5!y^1)_0 + \frac{x^6}{6!}(6!y^1)_0$$

$$+ \frac{x^7}{7!}(7!y_0)$$

$$y = y_0 + (y^1)_0 x + x^2y^1 + x^3y^1 + x^4y^1 + x^5y^1 + x^6y^1 + x^7y^1$$

$$= y_0 + y^1[x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7]$$

sub $y' = y_0$

$$y = y'[1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7]$$

$$y' = 0.0005$$

$$y = 0.0005\ [1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7]$$

(b) when $x = 5m$

$$y_5 = 0.0005\ [1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7]$$

$$= 0.0005\ [97656]$$

$$= 48.828$$

$$\simeq \underline{48.83}$$

When $x = 8m$

$$y_8 = 0.0005\ [1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7]$$

$$= 0.0005\ [2396745]$$

$$= 1118.3725$$

$$\simeq \underline{1198.37}$$

When $x = 10m$

$$y_{10} = 0.0005\ [1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7]$$

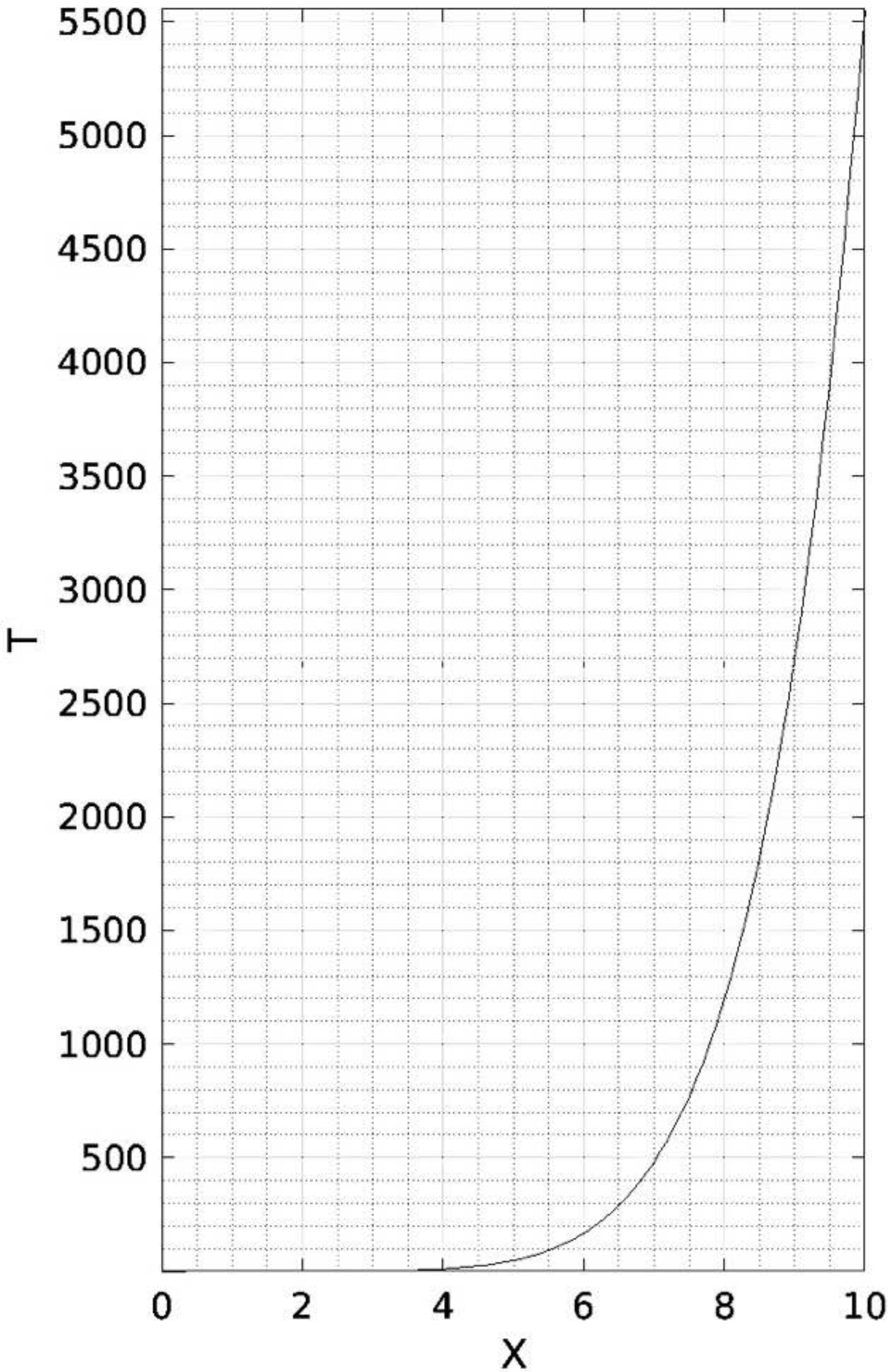
$$= 0.0005\ [11111111]$$

$$= 5555.5555$$

$$\simeq \underline{5555.56}$$

(c)

```
Command Window
Clear all
 Clc
 Close all
 Syms x Y
 x = 0:0.1:10
 Y = [(0.0005)*( 1+x+x.^2+x.^3+x.^4+x.^5 +x.^6+x.^7]
 Yn = Subs(Y)
 Ynn = double (Yn)
 Plot = (x, Ynn)
 x label = ('x')
 y label ('T')
 grid on
  grid minor
  axis tight
```


5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
T
0
2
4
6
8
10
X