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DEPARTMENT : COMPUTER ENGINEERING

The model for the deformation (y) of a structural element is represented by the expression given in Equation (1):

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Given that $y(0) = 0.0005m$ and $y'(0) = 0.0005$, applying Leibnitz-Maclaurin Method,

(a) obtain the power series solution of the model up to and including the term in x^7.

(b) estimate the approximate deformation when $x = 5, 8$ and $10m$ and

(c) with the aid of a MATLAB mfile program, plot the response of the structural element for $0 \leq x \leq 10m$.

(a) $x(x-1)y'' + (3x-1)y' + y = 0$

$= (x^2-x)y'' + (3x-1)y' + y = 0$

Applying Leibnitz theorem,

$$y^n = U^{(n)}V + nU^{(n-1)}V^{(1)} + \frac{n(n-1)U^{(n-2)}V^{(2)}}{2!} + \frac{n(n-1)(n-2)U^{(n-3)}V^{(3)}}{3!} + \dots$$

$W_1 = (x^2-x)y''$

$V = x^2 - x$ $\quad U^n = y'' = y^{(n+2)}$

$V^{(1)} = 2x - 1$ $\quad U^n = y^{(n+2)}$

$V^{(2)} = 2$ $\quad U^{n-1} = y^{(n+2)-1} = y^{n+1}$

$V^{(3)} = 0$ $\quad U^{n-2} = y^{(n+2)-2} = y^n$

$$W_1^n = y^{(n+2)}(x^2-x) + (n)y^{(n+1)}(2x-1) + \frac{n(n-1)y^n(2)}{2}$$

$$W_1^n = (x^2-x)y^{(n+2)} + (2x-1)(n)y^{(n+1)} + n(n-1)y^n$$

$$W_1^n = (x^2-x)y^{(n+2)} + (2nx-n)y^{(n+1)} + n(n-1)y^n$$

$W_2 = (3x-1)y'$

$V = 3x-1 \quad U^n = y' = y^{(n+1)}$

$V^{(1)} = 3 \quad U^n = y^{(n+1)}$

$V^{(2)} = 0 \quad U^{n-1} = y^{(n+1)-1} = y^n$

$W_2^n = y^{(n+1)}(3x-1) + n(y^n)(3)$

$W_2^n = (3x-1)y^{(n+1)} + 3n(y^{(n)})$

$W_2^n = (3x-1)y^{(n+1)} + 3n\, y^{(n)}$

$W_3 = y$

$V = 1 \quad U^n = y^n$

$V^{(1)} = 0$

$W_3^n = y^{(n)}(1) + (0) = y^n$

$W_1^n + W_2^n + W_3^n = 0$

$(x^2-x)y^{(n+2)} + (2nx-n)y^{(n+1)} + n(n-1)y^n + (3x-1)y^{(n+1)} + 3n\,y^{(n)} + y^n = 0$

$(x^2-x)y^{(n+2)} + (2nx-n+3x-1)y^{(n+1)} + (n^2-n+3n+1)y^n = 0$

$(x^2-x)y^{(n+2)} + (2nx-n+3x-1)y^{(n+1)} + (n^2+2n+1)y^n = 0$

At $x=0$,

$(2(0)x-n+3(0)-1)y^{(n+1)} + (n^2+2n+1)y^{(n)} = 0$

$(0-n+0-1)y^{(n+1)} + (n^2+2n+1)y^{(n)} = 0$

$-(n+1)y^{(n+1)} + (n^2+2n+1)y^{(n)} = 0$

$-(n+1)y^{(n+1)} = -(n^2+2n+1)y^{(n)}$

$(n+1)y^{(n+1)} = (n^2+2n+1)y^{(n)}$

$(n+1)y^{(n+1)} = (n+1)(n+1)y^{(n)}$

$y^{(n+1)} = \frac{(n+1)(n+1)}{(n+1)}y^{(n)}$

$y^{(n+1)} = (n+1)y^{(n)}$ (recurrence relation)

Applying the Maclaurin Series,

$$y = (y)_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \dots \frac{x^n (y^{(n)})_0}{n!} + \dots$$

At $n=0$, $y^{(0+1)}_0 = (y^{(1)})_0 = (0+1)y^{(0)} = (y)_0$

$n=1$, $y^{(1+1)}_0 = (y^{(2)})_0 = (1+1)y^{(1)} = 2(y^{(1)})_0$

$n=2$, $y^{(2+1)}_0 = (y^{(3)})_0 = (2+1)y^{(2)} = (3y^{(2)})_0 = (3)(2)(y^{(1)})_0 = 6(y^{(1)})_0$

$n=3$, $y^{(3+1)}_0 = (y^{(4)})_0 = (3+1)y^{(3)} = (4y^{(3)})_0 = (4)(3)(2)(y^{(1)})_0 = 24(y^{(1)})_0$

$n=4$, $y^{(4+1)}_0 = (y^{(5)})_0 = (4+1)y^{(4)} = (5y^{(4)})_0 = (5)(4)(3)(2)(y^{(1)})_0 = 120(y^{(1)})_0$

$n=5$, $y^{(5+1)}_0 = (y^{(6)})_0 = (5+1)y^{(5)} = (6y^{(5)})_0 = (6)(5)(4)(3)(2)(y^{(1)})_0 = 720(y^{(1)})_0$

$n=6$, $y^{(6+1)}_0 = (y^{(7)})_0 = (6+1)y^{(6)} = (7y^{(6)})_0 = (7)(6)(5)(4)(3)(2)(y^{(1)})_0 = 5040(y^{(1)})_0$

$$y = (y)_0 + x(y)_0 + \frac{x^2}{2!}(2)(y^{(1)})_0 + \frac{x^3}{3!}(6)(y^{(1)})_0 + \frac{x^4}{4!}(24)(y^{(1)})_0 + \frac{x^5}{5!}(120)(y^{(1)})_0$$

$$+ \frac{x^6}{6!}(720)(y^{(1)})_0 + \frac{x^7}{7!}(5040)(y^{(1)})_0$$

$$y = (1+x)(y)_0 + \frac{x^2}{2}(2)(y^{(1)})_0 + \frac{x^3}{6}(6)(y^{(1)})_0 + \frac{x^4}{24}(24)(y^{(1)})_0 + \frac{x^5}{120}(120)(y^{(1)})_0$$

$$+ \frac{x^6}{720}(720)(y^{(1)})_0 + \frac{x^7}{5040}(5040)(y^{(1)})_0$$

$y = (1+x)(y)_0 + x^2(y^{(1)})_0 + x^3(y^{(1)})_0 + x^4(y^{(1)})_0 + x^5(y^{(1)})_0 + x^6(y^{(1)})_0 + x^7(y^{(1)})_0$

$y = (1+x)(y)_0 + (y^{(1)}_0)(x^2+x^3+x^4+x^5+x^6+x^7)$ is the power series solution of the model.

(b) The approximate deformation when $x = 5m$, $8m$ and $10m$

(i) when $x = 5m$,

$y = (1+x)(y)_0 + (y^{(1)}_0)(x^2+x^3+x^4+x^5+x^6+x^7)$

Given that $(y)_0 = 0.0005m$, $(y')_0 = 0.0005$ and $x = 5m$

$y = (1+5)(0.0005m) + (0.0005)((5^2)m + (5^3)m + (5^4)m + (5^5)m + (5^6)m + (5^7)m)$

$y = (6)(0.0005m) + (0.0005)(25m + 125m + 625m + 3125m + 15625m + 78125m)$

$y = (3\times10^{-3}m) + (0.0005)(97650m) = (3\times10^{-3}m) + (48.825m)$

$y = 48.828m$

(ii) when $x = 8m$

$y = (1+x)(y)_0 + (y^{(4)})_0 (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$

$y = (1+8)(0.0005m) + 0.0005((8^2)m + (8^3)m + (8^4)m + (8^5)m + (8^6)m + (8^7)m)$

$y = (9)(0.0005m) + 0.0005(64m + 512m + 4096m + 32768m + 262144 + 2097152m)$

$y = (4.5\times10^{-3}m) + (0.0005)(2396736m)$

$y = (4.5\times10^{-3}m) + (1198.368m)$

$y = 1198.37m$

(iii) when $x = 10m$

$y = (1+x)(y)_0 + (y^{(4)})_0 (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$

$y = (1+10)(0.0005m) + 0.0005((10)^2m + (10^3)m + (10^4)m + (10^5)m + (10^6)m + (10^7)m)$

$y = (11)(0.0005m) + 0.0005(100m + 1000m + 10000m + 100000m + 1000000m + 10000000m)$

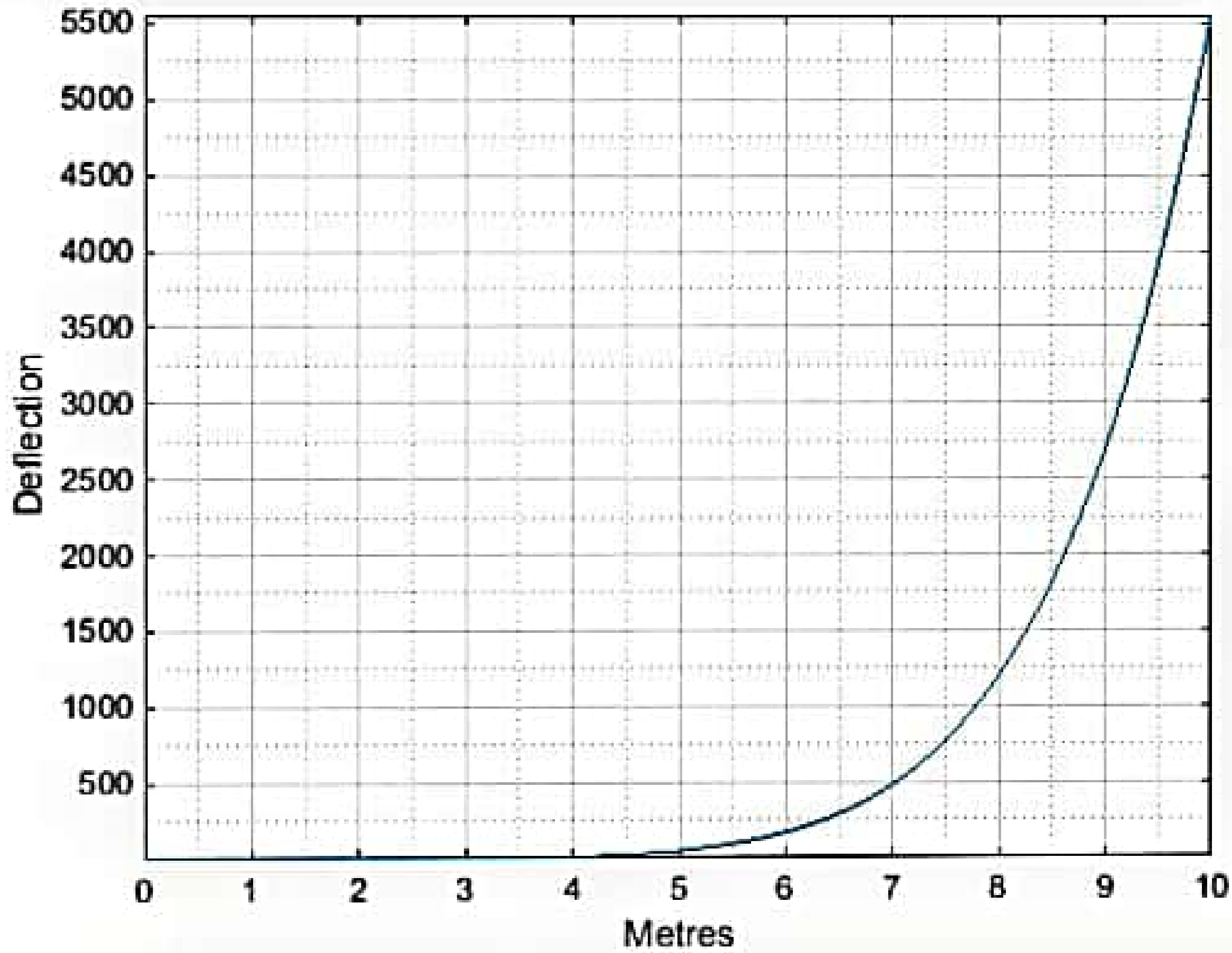
$y = (5.5\times10^{-3}m) + 0.0005(11111100m)$

$y = (5.5\times10^{-3}m) + (5555.55m)$

$y = 5555.55m$

(c) commandwindow

```
clear
clc
close all
syms x
x = 0:0.01:10
y =((0.0005)*(1+x))+((x.^2 + x.^3 + x.^4 + x.^5 + x.^6 + x.^7)*0.0005)
y1 = subs(y)
plot (x,y1)
xlabel('metres')
ylabel('Deflection')
grid on
grid minor
axis tight
```


5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
Deflection
0
1
2
3
4
5
6
7
8
9
10
Metres