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18/ENG06/005
Mechanical Engineering
Mathematics

(1) $y = 3e^{2x}$ and $y = 3e^{-x}$, $x = 1$, $x = 2$

For curve $y = 3e^{2x}$

$$A_1 = \int_1^2 y\,dx = \int_1^2 3e^{2x}\,dx$$

$$A_1 = \left[\frac{3}{2}e^{2x}\right]_1^2$$

$$A_1 = \frac{3}{2}\left[e^{2x}\right]_1^2 = \frac{3}{2}\left[e^{2\times 2} - e^{2\times 1}\right]$$

$$= \frac{3}{2}\left[e^4 - e^2\right] = \frac{3}{2}\left[47.2091\right]$$

$A_1 = 70.8137$ unit^2

For curve $y = 3e^{-x}$

$$A_2 = \int_1^2 y\,dx = \int_1^2 3e^{-x}\,dx$$

$$= 3\int_1^2 e^{-x}\,dx = -3\left[e^{-x}\right]_1^2$$

$$= -3\left[e^{-2} - e^{-1}\right] = -3\left[-0.2325\right]$$

$A_2 = 0.6975$ unit^2

• The area bounded by the curves $y = 3e^{2x}$ and $y =$

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$A_2 = \int_1^2 y\,dx = \int_1^2 3e^{-x}\,dx$

$= 3\int_1^2 e^{-x}\,dx = -3\left[e^{-x}\right]_1^2$

$= -3\left[e^{-2} - e^{-1}\right] = -3\left[-0.2325\right]$

$A_2 = 0.6975$ unit^2

$\therefore$ The area bounded by the curves $y = 3e^{2x}$ and $y = 3e^{-x}$ is

$A = A_1 - A_2$

$A = \left[70.8137 - 0.6975\right]$ unit^2

$A = 70.1162$ unit^2

(2) $y = 2\sin\frac{\pi}{10}t$ and $x = 2 + 2t - 2\cos\frac{\pi}{10}t$, $t = 0$ and $t = 10$

$A = \int_0^{10} y\,dx = \int_0^{10} 2\sin\frac{\pi}{10}t\,dx$

$x = 2 + 2t - 2\cos\frac{\pi}{10}t$

$\frac{dx}{dt} = 2 + \frac{\pi}{5}\sin\frac{\pi}{10}t$

$A_1 = \int_1^2 y\,dx = \int_1^2 8e^{2x}\,dx$

$A_1 = \left[\frac{8}{2} e^{2x}\right]_1^2$

$A_1 = \frac{8}{2}\left[e^{2x}\right]_1^2 = \frac{8}{2}\left[e^{2\times2} - e^{2\times1}\right]$

$= \frac{8}{2}\left[e^4 - e^2\right] = \frac{8}{2}\left[47.2091\right]$

$A_1 = 70.8187$ unit^2

For curve $y = 3e^{-x}$

$A_2 = \int_1^2 y\,dx = \int_1^2 3e^{-x}\,dx$

$= 3\int_1^2 e^{-x}\,dx = -3\left[e^{-x}\right]_1^2$

$= -3\left[e^{-2} - e^{-1}\right] = -3\left[-0.2325\right]$

$A_2 = 0.6975$ unit^2

$\therefore$ The area bounded by the curves $y = 8e^{2x}$ and $y = 3e^{-x}$ is

$A = A_1 - A_2$

$A = [70.8137 - 0.6975]$ unit^2

$A = 70.1162$ unit^2

(2) $y = 2\sin\frac{\pi}{10}t$ and $x = 2+2t-2\cos\frac{\pi}{10}t$, $t=0$ and $t=10$

$A = \int_0^{10} y\,dx = \int_0^{10} 2\sin\frac{\pi}{10}t\,dx$

$x = 2+2t-2\cos\frac{\pi}{10}t$

$\frac{dx}{dt} = 2 + \frac{\pi}{5}\sin\frac{\pi}{10}t$

$\therefore dx = \left(2 + \frac{\pi}{5}\sin\frac{\pi t}{10}\right)dt$

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therefore $A = \int_0^{10} \left(2 \sin \frac{\pi}{10} t\right) \left(2 + \frac{\pi}{5} \sin \frac{\pi}{10} t\right) dt$

$= \int_0^{10} \left(4 \sin \frac{\pi}{10} t + \frac{2\pi}{5} \sin^2 \frac{\pi}{10} t\right) dt$

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$= \int_0^{10} \left(4 \sin \frac{\pi}{10} t\right) dt + \int_0^{10} \left(\frac{2\pi}{5} \sin^2 \frac{\pi}{10} t\right) dt$

Integrating $\int_0^{10} \left(4 \sin \frac{\pi}{10} t\right) dt$ we have,

$\left[-\frac{40}{\pi} \cos \frac{\pi}{10} t\right]_0^{10}$

Integrating $\int_0^{10} \left(\frac{2\pi}{5} \sin^2 \frac{\pi}{10} t\right) dt$

From trigonometry; $\sin^2 x = \frac{1}{2} (1 - \cos 2x)$

therefore $\sin^2 \frac{\pi}{10} t = \frac{1}{2} \left(1 - \cos \frac{\pi}{5} t\right)$

then Integrating we have

$\frac{2\pi}{} \int \left(\sin^2 \frac{\pi}{} t\right) dt$

$A = 180^\circ$

$$\left[\frac{40}{\pi} + 2\pi - 0\right] - \left[\frac{-40}{\pi} + 0 - 0\right]$$

$$\frac{40}{\pi} + 2\pi + \frac{40}{\pi} = \left(\frac{80}{\pi} + 2\pi\right) \text{unit}^2$$

therefore $A = 25.4648 + 6.2832$

$A = 31.748\,\text{unit}^2$

$\left[-\frac{\quad}{\pi} \cos \frac{\quad}{10} \right]$

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From trigonometry; $\sin^2 x = \frac{1}{2}(1-\cos 2x)$

therefore $\sin^2 \frac{\pi}{10} t = \frac{1}{2}\left(1 - \cos\frac{\pi}{5} t\right)$

then Integrating we have

$$\frac{2\pi}{5} \int \left(\sin^2 \frac{\pi}{10} t\right) dt$$

$$\frac{2\pi}{5} \int \left(\frac{1}{2}\left(1 - \cos\frac{\pi}{5} t\right)\right) dt$$

$$\frac{2\pi}{10} \int 1 - \cos\frac{\pi}{5} t \, dt = \frac{2\pi}{10}\left[t - \frac{5}{\pi} \sin\frac{\pi}{5} t\right]$$

$$= \frac{\pi}{5}\left[t - \frac{5}{\pi} \sin\frac{\pi}{5} t\right]_0^{10}$$

therefore

$$\int_0^{10} \left(4 \sin\frac{\pi}{10} t + \frac{2\pi}{5} \sin^2 \frac{\pi}{10} t\right) dt =$$

$$\left[-\frac{40}{\pi} \cos\frac{\pi}{10} t + \frac{\pi}{5} t - \sin\frac{\pi}{5} t\right]_0^{10}$$

$$\left[-\frac{40}{\pi} \cos\frac{\pi}{10} t + \frac{\pi}{5} t - \sin\frac{\pi}{5} t\right]_0^{10} -$$

$$\left[-\frac{40}{\pi} \cos\frac{\pi}{10}(0) + \frac{\pi}{5}(0) - \sin\frac{\pi}{5}(0)\right]_0^{10}$$

$$= \int_0^{10} \left(4 \sin \frac{\pi}{10} t\right) dt + \int_0^{10} \left(\frac{2\pi}{5} \sin^2 \frac{\pi}{10} t\right) dt$$

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then Integrating we have

$$\frac{2\pi}{5} \int \left(\sin^2 \frac{\pi}{10} t\right) dt$$

$$\frac{2\pi}{5} \int \left(\frac{1}{2}\left(1 - \cos \frac{\pi}{5} t\right)\right) dt$$

$$\frac{2\pi}{10} \int 1 - \cos \frac{\pi}{5} t \; dt = \frac{2\pi}{10} \left[t - \frac{5}{\pi} \sin \frac{\pi}{5} t \right]$$

$$= \frac{\pi}{5} \left[t - \frac{5}{\pi} \sin \frac{\pi}{5} t \right]_0^{10}$$

therefore

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$$\left[-\frac{40}{\pi} \cos \frac{\pi}{10} t + \frac{\pi}{5} t - \sin \frac{\pi}{5} t \right]_0^{10}$$

$$\left[-\frac{40}{\pi} \cos \frac{\pi}{10} t + \frac{\pi}{5} t - \sin \frac{\pi}{5} t \right]^{10}$$

therefore $A = \int_0^{10} \left(2 \sin \frac{\pi}{10} t\right) \left(2 + \frac{\pi}{5} \sin \frac{\pi}{10} t\right) dt$

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$$\frac{2\pi}{5} \int \left(\sin^2 \frac{\pi}{10} t\right) dt$$

$$\frac{2\pi}{5} \int \left(\frac{1}{2}\left(1 - \cos \frac{\pi}{5} t\right)\right) dt$$

$$\frac{2\pi}{10} \int 1 - \cos \frac{\pi}{5} t \, dt = \frac{2\pi}{10} \left[t - \frac{5}{\pi} \sin \frac{\pi}{5} t\right]$$

$$\Big]^{10}$$