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MATRIC NO: 17/ENG06/037

DEPARTMENT: MECHANICAL ENGINEERING

Solution

$x(x-1)y'' + (3x-1)y' + y = 0$

Let

$W_1 = x(x-1)y''$

$W_2 = (3x-1)y'$

$W_3 = y$

for W_1

$u = y^2$ $\quad v = x(x-1) = x^2 - x$

$u^n = y^{n+2}$ $\quad v' = 2x - 1$

$u^{n-1} = y^{n+1}$ $\quad v'' = 2$

$u^{n-2} = y^n$ $\quad v''' = 0$

for W_2

$u = y'$ $\quad v = 3x - 1$

$u^n = y^{n+1}$ $\quad v' = 3$

$u^{n-1} = y^n$ $\quad v'' = 0$

For W_3

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v' = 0$

Recall

$$y = uv + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v^2 + \frac{n(n-1)(n-2)}{3!}u^{(n-3)}v^3 + \cdots$$

$$y = y^{(n+2)}.(x^2 - x) + n(y^{(n+1)})(2x-1) + \frac{n(n-1)}{2!}y^n.2 + \frac{n(n-1)(n-2)}{3!}(y^{(n-1)}).0 + (y^{(n+1)})(3x$$

$$+ ny^n.3 + \frac{n(n-1)}{2!}(y^{(n-1)}).0 + y^n.1 + n\,y^{(n-1)}.0$$

$$= (x^2 - x)(y^{(n+2)}) + n(2x-1)(y^{(n+1)}) + n(n-1)(y^{(n)}) + (3x-1)(y^{(n+1)}) + n\,3(y^n) + (y^n)$$

$$(x^2 - x)(y^{(n+2)}) + (y^{(n+1)}).(n(2x-1) + (3x-1)) + (y^{(n)}).(n(n-1) + 3n + 1)$$

$$= (x^2 - x).(y^{n+2}) + (y^{(n+1)}).(2nx - n + 3x - 1) + (y^n).(n^2 - n + 3n + 1)$$

assuming $x=0$

$(y^{(n+2)})_0(n+1)+(y^{(n)})_0(n^2+2n+1)=0$

$-(y^{(n+2)})_{(n+1)}+(y^{(n)})_0(n^2+2n+1)=0$

$(y^{(n+2)})_{(n+1)}=(y^{(n)})_0(n^2+2n+1)$

$(y^{(n+2)})_0=(y^{(n)})_0\frac{(n+1)(n+1)}{n+1}$

$(y^{(n+1)})_0=(y^{(n)})_0(n+1)$

Recall $(y^0)_0=0.0005$

$(y^{(0)})_0=0.0005$

$n=0$

$(y^{(0+1)})_0=(y^0)(0+1)$

$(y^{(1)})_0=1(y^0)_0$

$n=1$

$(y^{(1+1)})_0=(y^1)(1+1)$

$(y^2)_0=2(y^1)_0$

$n=2$

$(y^{(2+1)})_0=(y^2)(2+1)$

$(y^3)_0=3y^2=3\times(y^0)=6y^0$

$n=3$

$(y^{(3+1)})_0=(y^3)(3+1)$

$(y^4)_0=4(y^3)=4\times6(y^1)=24(y^1)_0$

$n=4$

$(y^{(4+1)})_0=(y^4)(4+1)$

$(y^5)_0=5(y^4)=5\times24(y^1)_0=120(y^1)_0$

$n=5$

$(y^{(5+1)})_0=(y^5)(5+1)$

$(y^6)_0=6(y^5)=6\times120(y^1)=720(y^1)_0$

$n=6$

$(y^{(6+1)})_0=(y^6)(6+1)$

$(y^7)_0=7(y^6)=7\times720(y^1)_0=5040(y^1)_0$

Using Leibnitz Maclaurin theory

$y=(y^{(0)})_0+x(y^1)_0+\frac{x^2(y^2)_0}{2!}+\frac{x^3(y^3)_0}{3!}+\frac{x^4(y^4)_0}{4!}+\frac{x^5(y^5)_0}{5!}+\frac{x^6(y^6)_0}{6!}+\frac{x^7(y^7)_0}{7!}$

$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^{(4)})_0 + \frac{x^5}{5!}(y^{(5)})_0 + \frac{x^6}{6!}(y^{(6)})_0 + \cdots$

$\cdots + \frac{x^9}{7!}(y^{(50400)})_0$

$y = (y^{(0)})_0 (1000) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)(y^{(1)})$

$y = 0.0005 (1+2) + 0.0005 (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$

When $x = 5$

$y = 0.0005 (1+5) + 0.0005 (5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)$

$= 11.8.825\ m$

$x = 8$

$y = 0.0005 (1+8) + 0.0005 (8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$

$= 11\ 98.3725\ m$

$x = 10$

$y = 0.0005 (1+10) + 0.0005 (10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$

$= 5555.5555\ m$

Practice file

Command window

clear

ac

close all

x = 0 : 0.01 : 10

y = (0.0005 * (1+x)) + (6x12 + x13 + x14 + x15 + x16 + x17) * 0.0005)

yn = Fiss4

Plot (x, yn)

xlabel ('m')

ylabel ('Deflection')

axis on

grid on

grid minor