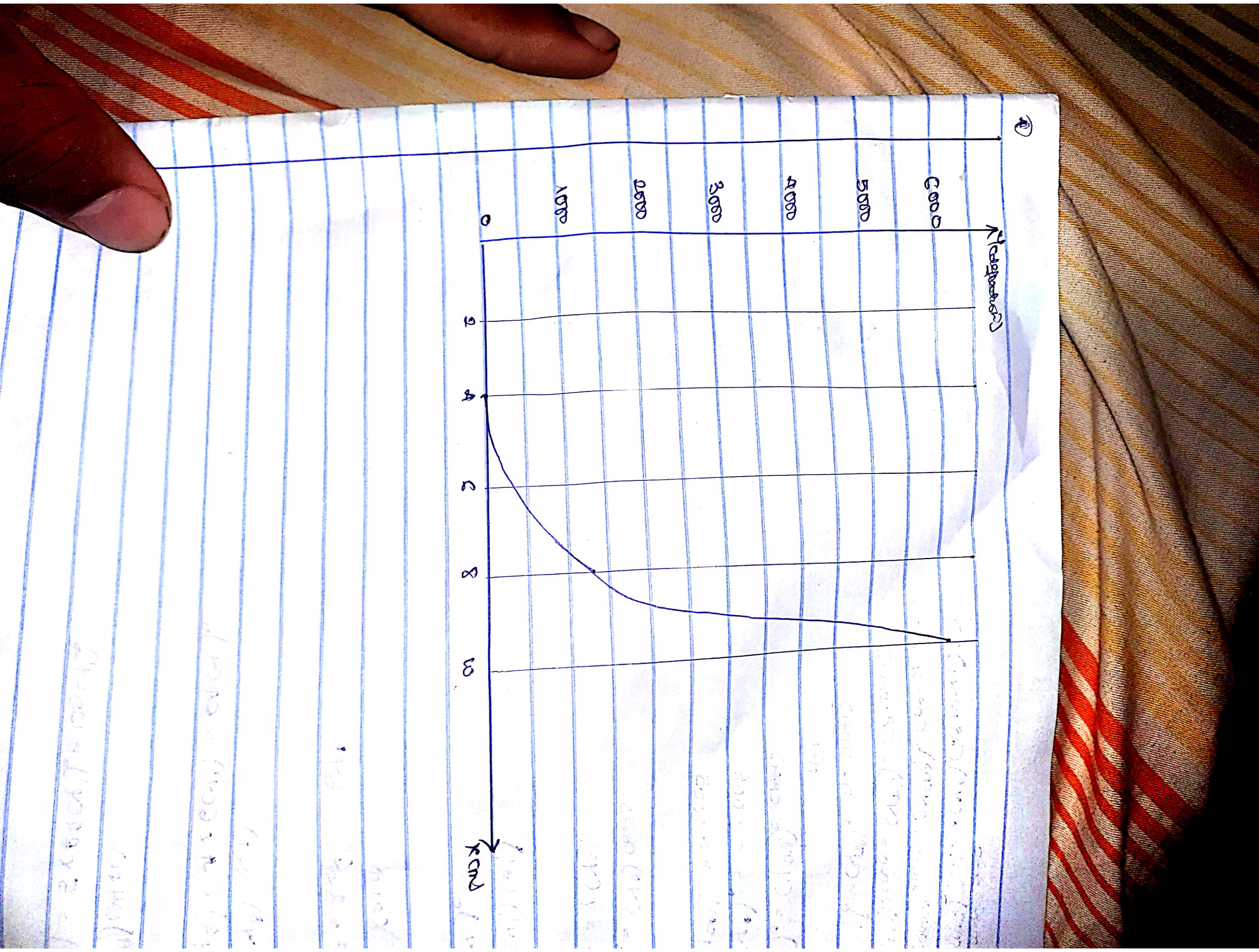
0
1000
2000
3000
4000
5000
6000
X(cm)

③

$y = (y^{(0)})_0 + x(y')_0 + \frac{x^2}{2!}(2y') + \frac{x^3}{3!}(6y') + \frac{x^4}{4!}(24y') + \frac{x^5}{5!}(120y') + \frac{x^6}{6!}(720y') + \ldots$

$\ldots + \frac{x^7}{7!}(5040y')_0$

$y = (y^{(0)})_0 (1+x) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)(y')$

$y = 0.0005(1+x) + 0.0005(x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$

when $x = 5$

$y = 0.0005(1+5) + 0.0005(5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)$

$= 48.828\,m$

$x = 8$

$y = 0.0005(1+8) + 0.0005(8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$

$= 1198.3725\,m$

$x = 10$

$y = 0.0005(1+10) + 0.0005(10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$

$= 5555.5555\,m$

Matlab file

Command window

clear

clc

close all

```
x = 0 : 0.01 : 10
y = (0.0005 * (1+x)) + ((x^2 + x^3 + x^4 + x^5 + x^6 + x^7) * 0.0005)
   yn = -5*y
 plot (x, yn)
xlabel ('m')
ylabel ('Deflection')
axis on
grid on
grid minor
```

(2)

assuming $x=0$

$(y^{(n+2)})_0 (-n-1) + (y^n)(n^2+2n+1) = 0$

$-(y^{(n+2)})(n+1) + (y^{(n)})_0 (n^2+2n+1) = 0$

$(y^{(n+2)}) \cdot (n+1) = (y^{(n)})_0 (n^2+2n+1)$

$(y^{(n+2)})_0 = (y^{(n)})_0 \dfrac{(n+1)(n+1)}{n+1}$

$(y^{(n+1)})_0 = (y^{(n)})_0 (n+1)$

Recall $(y^0)_0 = 0.0005$

$(y^{(1)})_0 = 0.0005$

$n=0$

$(y^{(0+1)})_0 = (y^0)(0+1)$

$(y^{(1)})_0 = 1(y^0)_0$

$n=1$

$(y^{(1+1)})_0 = (y')(1+1)$

$(y^2)_0 = 2(y')_0$

$n=2$

$(y^{(2+1)})_0 = (y^2)(2+1)$

$(y^3)_0 = 3y^2 = 3 \times 2(y') = 6y'$

$n=3$

$(y^{(3+1)})_0 = (y^3)(3+1)$

$(y^4)_0 = 4(y^3) = 4 \times 6(y') = 24(y')_0$

$n=4$

$(y^{(4+1)})_0 = (y^4)(4+1)$

$(y^5)_0 = 5(y^4) = 5 \times 24(y')_0 = 120(y')_0$

$n=5$

$(y^{(5+1)})_0 = (y^5)(5+1)$

$(y^6)_0 = 6(y^5) = 6 \times 120(y') = 720(y')_0$

$n=6$

$(y^{(6+1)})_0 = (y^6)(6+1)$

$(y^7)_0 = 7(y^6) = 7 \times 720(y')_0 = 5040(y')_0$

Using Leibnitz Maclaurin theory

$y = (y^{(0)})_0 + x(y')_0 + \dfrac{x^2}{2!}(y^2)_0 + \dfrac{x^3}{3!}(y^3)_0 + \dfrac{x^4}{4!}(y^4)_0 + \dfrac{x^5}{5!}(y^5)_0 + \dfrac{x^6}{6!}(y^6)_0 + \dfrac{x^7}{7!}(y^7)_0$

Scanned by CamScanner

Solution

$x(x-1)y'' + (3x-1)y' + y = 0$

Let

$W_1 = x(x-1)y''$

$W_2 = (3x-1)y'$

$W_3 = y$

for W_1

$u = y^2$ $\quad V = x(x-1) = x^2 - x$

$u^n = y^{n+2}$ $\quad V' = 2x - 1$

$u^{n-1} = y^{n+1}$ $\quad V'' = 2$

$u^{n-2} = y^n$ $\quad V''' = 0$

for W_2

$u = y'$ $\quad V = 3x - 1$

$u^n = y^{n+1}$ $\quad V' = 3$

$u^{n-1} = y^n$ $\quad V'' = 0$

For W_3

$u = y$ $\quad V = 1$

$u^n = y^n$ $\quad V' = 0$

Recall

$$y = u^n V + n u^{(n-1)} V' + \frac{n(n-1)}{2!} u^{(n-2)} V^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} V^3 + \dots$$

$$y = y^{(n+2)} \cdot (x^2 - x) + n(y^{(n+1)})(2x-1) + \frac{n(n-1)}{2!} y^n \cdot 2 + \frac{n(n-1)(n-2)}{3!} (y^{(n-1)}) \cdot 0 + (y^{(n+1)})(3x-1)$$

$$+ n y^n \cdot 3 + \frac{n(n-1)}{2!} (y^{(n-1)}) \cdot 0 + y^n \cdot 1 + n y^{(n-1)} \cdot 0$$

$= (x^2 - x)(y^{(n+2)}) + n(2x-1)(y^{(n+1)}) + n(n-1)(y^{(n)}) + (3x-1)(y^{(n+1)}) + n3(y^n)_0 (y^n)_0$

$(x^2 - x)(y^{(n+2)}) + (y^{(n+1)})_0 (n(2x-1) + (3x-1)) + (y^{(n)})_0 (n(n-1) + 3n + 1)$

$= (x^2 - x) \cdot (y^{(n+2)})_0 + (y^{(n+1)})_0 (2nx - n + 3x - 1) + (y^n)_0 (n^2 - n + 3n + 1)$