

Dunmoye Akrom O.

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Civil ENGR

ENG 382

Solution

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Letting  $x(x-1)y''' = w_1$

$$(3x-1)y' = w_2$$

$$y = w_3$$

Considering  $w_1$

$$U = y''$$

$$U' = y^{(n+2)}$$

$$U^{n-1} = y^{n+1}$$

$$U^{n-2} = y^n$$

$$V = x(x-1)$$

$$V' = 2x-1$$

$$V'' = 2$$

$$V''' = 0$$

~~Let~~

$$w_1^n = y^{n+2} \cdot (x^2 - x) + n y^{n+1} \cdot (2x-1) + n(n-1) y^n \cdot 2 + 0$$

$$w_1^n = y^{n+2} \cdot (x^2 - x) + n y^{n+1} (2x-1) + n y^n (n-1)$$

Considering  $w_2$

$$U = y'$$

$$U' = y^{(n+1)}$$

$$U^{n-1} = y^n$$

$$V = 3x-1$$

$$V' = 3$$

$$V'' = 0$$

$$w_2^n = y^{(n+1)} (3x-1) + n y^n \cdot 3 + 0$$

$w_3$

$$u = y \quad v = 1$$

$$u' = y^n \quad v' = 0$$

$$w_3^n = y^n - 1 + 0$$

$$w_3 = y^n$$

summation of  $w_1, w_2, w_3$

$$y^n = y^{n+2}(x^2 - 2x) + n y^{n+1}(2x - 1) + n y^n(n - 1) + y^{n+1}(3x - 1) + 3n y^n + y^n = 0$$

$$y^n = y^{n+2}(x^2 - 2x) + y^{n+1}(2xn - n) + y^n(n^2 - n) + y^{n+1}(3x - 1) + 3n y^n + y^n = 0$$

Collecting like terms

$$y^n = y^{n+2}(x^2 - 2x) + (2xn - 3x - n - 1)y^{n+1} + (n^2 - n + 3n + 1)y^n = 0$$

when  $x = 0$

$$0 = 0 + (0 - 0 - n - 1)y^{n+1} + (n^2 + 2n + 1)y^n = 0$$

$$-(n+1)y_0^{n+1} + (n^2 + 2n + 1)y_0^n = 0$$

$$-(n+1)y_0^{n+1} = -(n^2 + 2n + 1)y_0^n = -(n+1)(n+1)$$

$$y_0^{n+1} = \frac{-(n+1)(n+1)}{-(n+1)} y_0^n$$

$$y_0^{n+1} = (n+1)y_0^n$$

when  $n = 0$

$$y_0^{0+1} = 0+1 y_0^0$$

$$y_0^1 = y_0^0$$

when  $n = 1$

$$y_0^{n+1} = n+1 y_0^n$$

$$y_0^2 = 2y_0^1$$

when  $n = 2$

$$y_0^3 = 3y_0^2$$

$$= 3 \cdot 2y_0^1 = 6y_0^1$$

when  $n = 3$

$$y_0^4 = 4y_0^3$$

$$\Rightarrow 4 \cdot 3 \cdot 2y_0^1 = 24y_0^1$$

when  $n = 4$

$$y_0^5 = 5y_0^4$$

$$y_0^5 = 5 \cdot 4 \cdot 3 \cdot 2y_0^1 = 120y_0^1$$

when  $n = 5$

$$y_0^6 = 6y_0^5$$

$$y_0^6 = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2y_0^1 = 720y_0^1$$

$$y = (y_0') + x(y_0') + \frac{x^2}{2!} y_0'' + \frac{x^3}{3!} (y_0''') + \frac{x^4}{4!} y_0^{(4)} + \frac{x^5}{5!} (y_0^{(5)}) + \frac{x^6}{6!} (y_0^{(6)})$$

$$y = y_0' + x(y_0') + \frac{x^2}{2!} 2y_0'' + \frac{x^3}{3!} 6y_0''' + \frac{x^4}{4!} 24y_0^{(4)} + \frac{x^5}{5!} 120y_0^{(5)} + \frac{x^6}{6!} 720y_0^{(6)}$$

$$y = 6(x+1)y_0' + (x^2 + x^3 + x^4 + x^5 + x^6)y_0''$$

$$y_0' = 0.0005 \text{ m}$$

$$y_0'' = 0.0005$$

$$y = 6(x+1)0.0005 \text{ m} + (x^2 + x^3 + x^4 + x^5 + x^6)0.0005$$

$$\text{when } x = 5$$

$$y = 6(0.0005) + (5^2 + 5^3 + 5^4 + 5^5 + 5^6)0.0005$$

$$y = 9.7655 \text{ m}$$

$$\text{when } x = 8$$

$$y = 9(0.0005) + (8^2 + 8^3 + 8^4 + 8^5 + 8^6)0.0005$$

$$y = 149.7965 \text{ m}$$

$$\text{when } x = 10 \text{ m}$$

$$y = 11(0.0005) + (10^2 + 10^3 + 10^4 + 10^5 + 10^6)0.0005$$

$$y = 555.56 \text{ m}$$



# MATLAB CODE

command window

clear

clc

close all

syms x

$$y = ((1+x)^5 * (0.0005)) + (x^2 + x^3 + x^4 + x^5 + x^6) * (0.0005)$$

t = 0:0.1:10

xt = subs(y,t)

xt\_n = double(xt)

plot(t, xt\_n)

xlabel('t')

ylabel('x')

grid on

grid minor

axis tight

