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H/ENQ03/037

Civil Engineering, 300L

ENQ 382, Engineering Maths, III

Assignment III

Solution

$$x(x-1)y'' + (3x-1)y' + y = 0$$

$$W_1 = x(x-1)y''$$

$$W_2 = (3x-1)y'$$

$$W_3 = y$$

For W_1 ,

$$v = x(x-1) \quad v' = 2x-1 \quad v'' = 2 \quad v''' = 0$$

$$u = y'' \quad u' = y''' \quad u'' = y^{(4)} \quad u''' = y^{(5)}$$

$$u^n = y^{n+2}$$

$$W_1^n = y^{n+2} (x^2-x) + ny^{n+1} (2x-1) + n(n-1)y^n \cdot 2 + 0$$

For W_2 ,

$$v = 3x-1 \quad v' = 3 \quad v'' = 0$$

$$u = y' \quad u' = y'' \quad u'' = y'''$$

$$u^n = y^{n+1}$$

$$W_2^n = y^{n+1} \cdot 3x-1 + ny^n \cdot (3) + (y^{n-1}) \cdot 0$$

For W_3 ,

$$v = 1 \quad v' = 0$$

$$u = y \quad u' = y'$$

$$u^n = y^n$$

$$W_3^n = y^n \cdot 1$$

$$W_1 + W_2 + W_3 = 0$$

$$y^{n+2} (x^2-x) + ny^{n+1} (2x-1) + ny^n (n-1) + y^{n+1} (3x-1) + 3ny^n + y^n = 0$$

$$y^{n+2} (x^2-x) + y^{n+1} (2x-3x-n-1) + y^n (n^2-n+3n+1) = 0$$

where $x=0$

$$-(n+1)y_0^{n+1} + (n^2 + 2n+1)y_0^n = 0$$

$$-(n+1)y_0^{n+1} = -(n^2 + 2n+1)y_0^n$$

$$y_0^{n+1} = n+1 y_0^n$$

When $n=0$

$$y_0^1 = y_0^0$$

when $n=1$

$$y_0^2 = 2y_0^1$$

when $n=2$

$$y_0^3 = 3y_0^2$$

$$= 3(2y_0^1) = 6y_0^1$$

~~when~~ when $n=3$

$$y_0^4 = 4y_0^3$$

$$= 4(6y_0^1) = 24y_0^1$$

when $n=4$

$$y_0^5 = 5y_0^4$$

$$= 5(24y_0^1) = 120y_0^1$$

when $n=5$

$$y_0^6 = 6y_0^5$$

$$= 6(120y_0^1) = 720y_0^1$$

$$y = y_0^0 + x(y_0^1) + \frac{x^2}{2!}y_0^2 + \frac{x^3}{3!}y_0^3 + \frac{x^4}{4!}y_0^4 + \frac{x^5}{5!}y_0^5 + \frac{x^6}{6!}y_0^6$$

$$y = y_0^0 + x(y_0^1) + \frac{x^2}{2!}2y_0^1 + \frac{x^3}{6}6y_0^1 + \frac{x^4}{24}24y_0^1 + \frac{x^5}{120}120y_0^1 + \frac{x^6}{720}720y_0^1$$

$$y = y_0^0(x+1) + y_0^1(x^2+x^3+x^4+x^5+x^6)$$

$$\text{when } y_0^0 = 0.0005 \text{ m}$$

$$y_0^1 = 0.0005 \text{ m} \quad \xi_1 \quad x=5$$

$$y = 6(0.0005) + (0.0005)(5^2+5^3+5^4+5^5+5^6)$$

$$y = 9.7655 \text{ m}$$

$$\text{when } y_0^0 = 0.0005 \text{ m}$$

$$y_0^1 = 0.0005 \text{ m} \quad \xi_1 \quad x=8$$

$$y = 9(0.0005) + (8^2+8^3+8^4+8^5+8^6)(0.0005)$$

$$y = 149.7965 \text{ m}$$

$$\xi_1 \quad x=10$$

$$y = 585.56 \text{ m}$$

MATLAB

command window

clear

clc

close all

syms x

$$y = ((1+x) * (0.0005)) + ((x^2 + x^3 + x^4) * x^5 + x^6) * (0.0005)$$

x = 0:0.1:10

xt = subs(x)

yt = double(yt)

plot(x, yt)

xlabel('t')

ylabel('n')

grid on

grid minor

axis tight

GRAPH of 't' against 'n'

