AGWANIRU ROSEMARY ONYINYECHI
17/ENG01/003
CHEMICAL ENGINEERING
ENG 381 SOLUTION TO ASSIGNMENT 3

Given

$x(x-1)y'' + (3x-1)y' + y = 0$

Applying leibnithz theorem.

$(x^2-x)y'' + (3x-1)y' + y = 0$

let

$a_1 = (x^2-x)y''$

$a_2 = (3x-1)y'$

$a_3 = y'$

Finding the nth derivative for them.

a_1

$u = y''$ $\qquad$ $v = x^2 - x$

$u^n = y^{(n+2)}$ $\qquad$ $v' = 2x-1$

$u^{(n-1)} = y^{(n+1)}$ $\qquad$ $v'' = 2$

$u^{(n-2)} = y^n$ $\qquad$ $v''' = 0$

$\therefore\ a_1^{(n)} = y^{(n+2)}.(x^2-x) + n\,y^{(n+1)}.(2x-1) + \frac{n(n-1)}{2!}y^n.2 + 0$

a_2

$u = y'$ $\qquad$ $v = (3x-1)$

$u^n = y^{(n+1)}$ $\qquad$ $v' = 3$

$u^{(n-1)} = y^n$ $\qquad$ $v'' = 0$

$a_2^{(n)} = y^{(n+1)}.(3x-1) + ny^n.3$

a_3

$u = y$ $\qquad$ $v = 1$

$u^n = y^n$ $\qquad$ $v' = 0$

$a_3^{(n)} = y^n.1$

$\therefore\ y^{(n+2)}.(x^2-x) + ny^{(n+1)}(2x-1) + n(n-1)y^n + y^{(n+1)}.(3x-1) + ny^n.3$
$+ y^n = 0$

Assuming $x=0$

$$n(n-1)y^{n}+3ny^{n}+y^{n}-ny^{(n+1)}-y^{(n+1)}=0$$

$$(n^2+2n+1)y^{n}-(n+1)y^{(n+1)}=0$$

$$y^{(n+1)}=\frac{n^2+2n+1}{n+1}y^{n}$$

$$\boxed{y^{(n+1)}=(n+1)y^{n}}$$ * Recurrent Relation.

when $n=0$

$$(y^{(1)})_0=(y^{0})_0$$

when $n=1$

$$(y^{(2)})_0=2(y^{(1)})_0=2(y^{(0)})_0$$

when $n=2$

$$(y^{(3)})_0=3(y^{(2)})=3\times2(y^{(0)})_0$$

when $n=3$

$$(y^{(4)})_0=4(y^{(3)})=4\times3\times2(y^{(0)})_0$$

when $n=4$

$$(y^{(5)})_0=5(y^{(4)})_0=5\times4\times3\times2(y^{(0)})_0$$

when $n=5$

$$(y^{(6)})_0=6(y^{(5)})_0=6\times5\times4\times3\times2(y^{(0)})_0$$

when $n=6$

$$(y^{(7)})_0=7(y^{(6)})_0=7\times6\times5\times4\times3\times2(y^{(0)})_0$$

Applying Maclaurin series.

$$y=(y^{(0)})_0+x(y^{(1)})_0+\frac{x^2}{2!}(y^{(2)})_0+\frac{x^3}{3!}(y^{(3)})_0+\frac{x^4}{4!}(y^{(4)})_0+\cdots$$

$$y = \left(y^{(0)}\right)_0 + x\left(y^{(0)}\right)_0 + \frac{x^2}{2!}\,2\left(y^{(0)}\right)_0 + \frac{x^3}{3!}\times 3\times 2\left(y^{(0)}\right)_0 + \frac{x^4}{4!}\cdot 4\times 3\times 2\times\left(y^{(0)}\right)_0$$

$$+ \frac{x^5}{5!}\cdot 5\times 4\times 3\times 2\left(y^{(0)}\right)_0 + \frac{x^6}{6!}\times 6\times 5\times 4\times 3\times 2\left(y^{(0)}\right)_0 + \frac{x^7}{7!}\,7\times 6\times 5\times 4\times 3\times 2\left(y^{(0)}\right)_0$$

$$+ \cdots\cdots$$

$$y = \left(y^{(0)}\right)_0 + x\left(y^{(0)}\right)_0 + x^2\left(y^{(0)}\right)_0 + x^3\left(y^{(0)}\right)_0 + x^4\left(y^{(0)}\right)_0 + x^5\left(y^{(0)}\right)_0$$

$$+ x^6\left(y^{(0)}\right)_0 + x^7\left(y^{(0)}\right)_0 + \cdots$$

$$y = \left(y^{(0)}\right)_0\left[1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \cdots\right)$$

a) For power series.

Since $\left(y^{(0)}\right)_0 = 0.0005\,m$ and $\left(y^{(1)}\right)_0 = 0.0005\,m$

$$y = 0.0005\left(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \cdots\right)$$

b) when $x = 5\,m$

$$y = 0.0005\left(1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7 + 5 \cdots\right)$$

$$y = 48.828\,m = 48.828\,m$$

when $x = 8\,m$

$$y = 0.0005\left(1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7 + \cdots\right)$$

$$y = 1198.3125\,m$$

when $x = 10\,m$

$$y = 0.0005\left(1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7\right)$$

$$y = 5555.5556\,m$$

Using Matlab

- Commandwindow
- clear
- clc
- close all
- x = 0:0.01:10
- y = 0.0005 * (1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)
- yn = subs(y)
- Plot (x, yn)
- xlabel ('m')
- ylabel ('Deflection')
- axis tight
- grid on
- grid minor.