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17/ENG01/030

CHEMICAL ENGINEERING

ENG 381 ASSIGNMENT 3.

$x(x-1)y'' + (3x-1)y' + y = 0$

Applying Leibnitz - Maclaurin method

$(x^2 - x)y'' + (3x-1)y' + y = 0$

$(x^2 - x)y^{(2)} + (3x-1)y' + y = 0$

Find the nth derivative

Let $(x^2 - x)y^{(2)} - W_1$

$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)u^{(n-2)} v''}{2!} + \ldots$

$u = y^2$ $\quad v = x^2 - x$

$u^n = y^{n+2}$ $\quad v' = 2x - 1$

$u^{n-1}\ y^{n+1}$ $\quad v'' = 2$

$u^{n-2} = y^n$ $\quad v''' = 0$

$W_1^{(n)} = y^{n+2} - x^2 - x + n y^{n+1} \cdot 2x - 1 + \frac{n(n-1)}{2!} y^n \cdot 2$

Let $(3x-1)y' = W_2$

$u = y'$ $\quad v = 3x - 1$

$u^n = y^{n+1}$ $\quad v' = 3$

$u^{n-1} = y^n$ $\quad v'' = 0$

$W_2^n = y^{n+1} \cdot 3x - 1 + n y^n \cdot 3$

Let $y = W_3$

$W_3^{(n)} = y^n$

nth derivative

$y^{n+2} \cdot (x^2 - x] + n y^{n+1} \cdot 2x - 1 + n(n-1) y^n + y^n + 3x + n y^n \cdot 3 + y^n = 0$

Assuming $x = 0$

$n(n-1)y^n + 3n y^n + y^n - n y^{n+1} - y^{n+1} = 0$

$(n^2 + 2n + 1)y^n - (n+1)\, y^{(n+1)} = 0$

$= \frac{n^2 + 2n + 1\ y^n}{n+1} = \frac{(n+1)(\cancel{n+1})y^n}{(\cancel{n+1})}$

$y^{n+1} = (n+1)y^n$ → Recurrent relation.

when $n=0$ $(y^1)_0 = (y^1)_0$

$n=1$ $(y^2)_0 = 2(y^1)_0 = 2(y^0)_0$

$n=2$ $(y^3)_0 = 3(y^2)_0 = 3\times2(y^0)_0$

$n=3$ $(y^4)_0 = 4(y^3)_0 = 4\times3\times2(y^0)_0$

$n=4$ $(y^5)_0 = 5(y^4)_0 = 5\times4\times3\times2(y^0)_0$

$n=5$ $(y^6)_0 = 6(y^5)_0 = 6\times5\times4\times3\times2(y^0)_0$

$n=6$ $(y^7)_0 = 7(y^6) = 7\times6\times5\times4\times3\times2(y^0)_0$

Applying Maclaurins Series.

$$y = (y^0)_0 + x(y^1)_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \frac{x^5}{5}(y^5)_0$$

$$y = (y^0)_0 + x(y^0)_0 + \frac{x^2}{2!}\times 2(y^0)_0 + \frac{x^3}{3!}(3\times2)(y^3)_0 + \frac{x^4}{4!}4\times3\times2(y^4)_0$$

$$\frac{x^5}{5!}5\times4\times3\times2(y^5) + \frac{x^6}{6!}6\times5\times4\times3\times2(y^6)_0 + \frac{x^7}{7!}7\times6\times5\times4\times3\times2(y^7)_0$$

$$\therefore y = (y^0)_0 + x(y^0)_0 + x^2(y^0)_0 + x^3(y^0)_0 + x^4(y^0)_0 + x^5(y^0)_0 + x^6(y^0)_0 + x^7(y^0)_0$$

Power series

$$y = (y^0)_0(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

Since $(y^0)_0 = 0.0005(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$

b. When $x = 5$

$$y = 0.0005(1 + 5 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)$$

$y = 48.828m$

When $x = 8$

$$y = 0.0005(1 + 8 + 8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$$

$y = 1198.3725m$

when $x = 10m$

$y = 0.0005(1 + 10 + 10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$

$y = 5555.556m.$

(C.) MATLAB mfule.

- Command window
- clear
- clc
- close all
- x = 0 : 0.1 : 10
- y = 0.005 + (1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)
- y^ = subs (y)
- plot (x, yn)
- x label ('m')
- y label ('Deflection')
- axis tight
- grid on
- grid minor.