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17/ENG01/008

CHEMICAL ENGINEERING

ENG 381 — ENGINEERING MATHEMATICS 3 ASSIGNMENT 3

① Question: The model for the deformation (y) of a structural element is represented by the expression given in equation (1):

$$x(x-1)y'' + (3x-1)y' + y = 0 \quad \text{——} (1)$$

Given that $y(0) = 0.0005m$ & $y'(0) = 0.0005$, applying Leibnitz-Maclaurin Method

(a) Obtain the power series solution of the model up to and including the term in x^7;

(b) Estimate the approximate deformation when $x = 5, 8$ and $10m$ and

(c) With aid of a MATLAB mfile program, plot the response of the structural element for $0 \le x \le 10m$.

SOLUTION

(a) $$x(x-1)y'' + (3x-1)y' + y = 0$$

The Equation is rewritten as:

$$(x-1)y^{(2)} + (3x-1)y^{(1)} + y = 0$$

$W_1 = x(x-1)y^{(2)}$

$u = y^{(2)}$ $\qquad$ $v = (x-1)x = x^2 - x$

$u^{(n)} = y^{(n+2)}$ $\qquad$ $v^{(1)} = 2x - 1$

$u^{(n-1)} = y^{(n+1)}$ $\qquad$ $v^{(2)} = 2$

$u^{(n-2)} = y^{(n)}$ $\qquad$ $v^{(3)} = 0$

Applying Leibnitz Equation:

$$y^{(n)} = u^{(n)}v + nu^{(n-1)}v^{(1)} + \frac{n(n-1)}{2!}u^{(n-2)}v^{(2)} + \frac{n(n-1)(n-2)}{3!}u^{(n-3)}v^{(3)}$$

$$+ \frac{n(n-1)(n-2)(n-3)}{4!}u^{(n-4)}v^{(4)} + \dots$$

$$y^{(n)} = y^{(n+2)} \times x(x-1) + n \times y^{(n+1)} \times (2x-1) + \frac{n(n-1)}{\cancel{2}} \times y^{(n)} \times \cancel{2} + 0$$

$$y^{(n)} = x(x-1)y^{(n+2)} + (2x-1)ny^{(n+1)} + n(n-1)y^{(n)}$$

For $W_2 = (3x-1)y^{(1)}$

$u = y^{(1)}$ $\qquad$ $v = 3x - 1$

$u^{(n)} = y^{(n+1)}$ $\qquad$ $v^{(1)} = 3$

$u^{(n-1)} = y^{(n)}$ $\qquad$ $v^{(2)} = 0$

Applying Leibnitz theorem

$y^{(n)} = y^{(n+1)} \times (3x-1) + n \times y^{(n)} \times 3 + 0$

$y^{(n)} = (3x-1) y^{(n+1)} + 3n y^{(n)}$

For $W_3 = y$

$u = y$ $\quad v = 1$

$u^{(n)} = y^{(n)}$ $\quad v^{(1)} = 0$

Applying Leibnitz theorem:

$y^{(n)} = y^{(n)} \times 1 + 0 = y^{(n)}$

Summing all equations together;

$x(x-1) y^{(n+2)} + (2x-1) n y^{(n+1)} + n(n-1) y^{(n)} + (3x-1) y^{(n+1)} + 3n y^{(n)} + y^{(n)} = 0$

Assuming $x = 0$

$-n(y^{(n+1)})_0 + n(n-1)(y^{(n)})_0 - (y^{(n+1)})_0 + 3n(y^{(n)})_0 + (y^{(n)})_0 = 0$

$-(y^{(n+1)})_0 [n+1] + (y^{(n)})_0 [n(n-1)+3n+1] = 0$

$-(y^{(n+1)})_0 (n+1) + (y^{(n)})_0 (n^2 - n + 3n + 1) = 0$

$-(y^{(n+1)})_0 (n+1) + (y^{(n)})_0 (n^2 + 2n + 1) = 0$

$+(n+1)(y^{(n+1)})_0 = +(n^2 + 2n + 1)(y^{(n)})_0$

$(n+1)(y^{(n+1)})_0 = (n^2 + 2n + 1)(y^{(n)})_0$

When $n = 0$

$(y^{(1)})_0 = (y^{(0)})_0$

But $(y^{(0)})_0 = 0.0005m$

$\therefore (y^{(1)})_0 = 0.0005$

When $n = 1$

$2(y^{(2)})_0 = (1 + 2 + 1)(y^{(1)})_0$

$2(y^{(2)})_0 = 4(y^{(1)})_0$; $(y^{(2)})_0 = 2(y^{(1)})_0 = 2 \times 0.0005 = 1 \times 10^{-3}$

When $n = 2$

$\cancel{3}(y^{(3)})_0 = (4+4+1)(y^{(2)})_0 = \frac{3}{\cancel{9}}(y^{(2)})_0$; $(y^{(3)})_0 = 3(y^{(2)})_0$

But $(y^{(2)})_0 = 1 \times 10^{-3}$; $(y^{(3)})_0 = 3 \times 1 \times 10^{-3} = 3 \times 10^{-3}$

When $n = 3$; $4(y^{(4)})_0 = (9+6+1)(y^{(3)})_0 = 16(y^{(3)})_0$

$(y^{(4)})_0 = 4(y^{(3)})_0 = 4 \times 3 \times 10^{-3} = 0.012$

When $n=4$; $5(y^{(5)})_0 = (16+8+1)(y^{(4)})_0 = 25(y^{(4)})_0 = 25\times 0.012 = 0.3$

$(y^{(5)})_0 = 0.06$

When $n=5$; $6(y^{(6)})_0 = (25+10+1)(y^{(5)})_0 = 36(y^{(5)})_0 = 6(y^{(5)})_0$

$(y^{(6)})_0 = 6(y^{(5)})_0 = 6\times 0.06 = 0.36$

When $n=6$; $7(y^{(7)})_0 = (36+12+1)(y^{(6)})_0 = 49(y^{(6)})_0$

$(y^{(7)})_0 = 7(y^{(6)})_0 = 7\times 0.36 = 2.52$

Leibnitz Maclaurins Method Equation

$$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^{(4)})_0 + \frac{x^5}{5!}(y^{(5)})_0$$
$$+ \frac{x^6}{6!}(y^{(6)})_0 + \frac{x^7}{7!}(y^{(7)})_0$$

$$y = 0.0005 + 0.0005x + \frac{x^2}{2}\times 1\times10^{-3} + \frac{x^3}{6}\times 3\times10^{-3} + \frac{x^4}{24}\times 0.012 + \frac{x^5}{120}\times 0.06$$
$$+ \frac{x^6}{720}\times 0.36 + \frac{x^7}{5040}\times 2.52$$

$$y = 0.0005(1+x) + 5\times10^{-4}(x^2) + 5\times10^{-4}x^3 + 5\times10^{-4}x^4 + 5\times10^{-4}x^5 + 5\times10^{-4}x^6$$
$$+ 5\times10^{-4}x^7$$

$$y = 0.0005(1+x) + 5\times10^{-4}(x^2+x^3+x^4+x^5+x^6+x^7)$$

$$y = 5\times10^{-4}(1+x) + 5\times10^{-4}(x^2+x^3+x^4+x^5+x^6+x^7)$$

$$y = 5\times10^{-4}[(1+x) + (x^2+x^3+x^4+x^5+x^6+x^7)]$$

$$y = 5\times10^{-4}[1+x+x^2+x^3+x^4+x^5+x^6+x^7]$$

(b)

→ When $x=5$

$y = 5\times10^{-4}[1+5+25+125+625+3125+15625+78125]$

$y = 5\times10^{-4}\times 97656$

$y = 48.828$

→ When $x=8$

$y = 5\times10^{-4}[1+8+64+512+4096+32768+262144+2097152]$

$y = 1198.3725$

→ When $x=10$

$y = 5\times10^{-4}(1+10+100+1000+10000+10^5+10^6+10^7) = 5555.5555$

(c) Codes:

```
Commandwindow
clear
clc
close all
syms x
Y=(5*(10.^-4)*(1+x+x.^2+x.^3+x.^4+x.^5+x.^6+x.^7)
x=(0:10)
Yn=subs(Y,x)
Plot(x,Yn)
xlabel('STRUCTURAL ELEMENT (m)')
Ylabel('DEFORMATION')
grid on
grid minor
axis tight
```