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MAT NO: 17/ENG03/028
DEPT: CIVIL ENGINEERING
ASSIGNMENT 3

$x(x-1)y'' + (3x-1)y' + y = 0$

Let

$W_1 = x(x-1)y''$

$W_2 = (3x-1)y'$

$W_3 = y$

for W_1

$u = y''$ $\quad v = x(x-1) = x^2 - x$

$u^{(n)} = y^{(n+2)}$ $\quad v' = 2x - 1$

$u^{(n-1)} = y^{(n+1)}$ $\quad v'' = 2$

$u^{(n-2)} = y^{(n)}$ $\quad v''' = 0$

for W_2

$u = y'$ $\quad v = 3x - 1$

$u^{(n)} = y^{(n+1)}$ $\quad v' = 3$

$u^{(n-1)} = y^{(n)}$ $\quad v'' = 0$

For W_3

$u = y$ $\quad v = 1$

$u^{(n)} = y^{(n)}$ $\quad v' = 0$

Recall

$$y = uv + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v^2 + \frac{n(n-1)(n-2)}{3!}u^{(n-3)}v^3 + \dots$$

$$y = y^{(n+2)}(x^2 - x) + n(y^{(n+1)})(2x-1) + \frac{n(n-1)}{2!}y^{(n)}\cdot 2 + \frac{n(n-1)(n-2)}{3!}(y^{(n-1)})\cdot 0 + (y^{(n)})(3x-1)$$

$$+ ny^{(n)}\cdot 3 + n(n-1)(y^{(n-1)})\cdot 0 + y^{(n)}\cdot 1 + n y^{(n-1)}\cdot 0$$

$$= (x^2 - x)(y^{(n+2)}) + n(2x-1)(y^{(n+1)}) + n(n-1)(y^{(n)}) + (3x-1)(y^{(n+1)}) + n3(y^{(n)}) + (y^{(n)})$$

$$(x^2 - x)(y^{(n+2)}) + (y^{(n+1)})\cdot(n(2x-1) + (3x-1)) + (y^{(n)})\cdot(n(n-1) + 3n + 1)$$

$$= (x^2 - x)\cdot(y^{(n+2)}) + (y^{(n+1)})\cdot(2nx - n + 3x - 1) + (y^{(n)})\cdot(n^2 - n + 3n + 1)$$

assuming $x=0$

$(y^{(n+2)})_0 (n+1) + (y^{(n)})_0 (n^2+2n+1) = 0$

$-(y^{(n+2)})_0 (n+1) + (y^{(n)})_0 (n^2+2n+1) = 0$

$(y^{(n+2)})_0 \cdot (n+1) = (y^{(n)})_0 (n^2+2n+1)$

$(y^{(n+2)})_0 = (y^{(n)})_0 \frac{(n+1)(n+1)}{n+1}$

$(y^{(n+2)})_0 = (y^{(n)})_0 (n+1)$

Recall $(y^0)_0 = 0.0005$

$(y^{(0)})_0 = 0.0005$

$n=0$

$(y^{(0+2)})_0 = (y^0)_0 (0+1)$

$(y^{(2)})_0 = 1 (y^0)_0$

$n=1$

$(y^{(1+2)})_0 = (y^1)_0 (1+1)$

$(y^2)_0 = 2(y^1)_0$

$n=2$

$(y^{(2+2)})_0 = (y^2)_0 (2+1)$

$(y^3)_0 = 3y^2 = 3 \times 2(y^1)_0 = 6y^1$

$n=3$

$(y^{(3+1)})_0 = (y^3)_0 (3+1)$

$(y^4)_0 = 4(y^3) = 4 \times 6(y^1) = 24(y^1)_0$

$n=4$

$(y^{(4+1)})_0 = (y^4)_0 (4+1)$

$(y^5)_0 = 5(y^4) = 5 \times 24(y^1)_0 = 120(y^1)_0$

$n=5$

$(y^{(5+1)})_0 = (y^5)_0 (5+1)$

$(y^6)_0 = 6(y^5) = 6 \times 120(y^1) = 720(y^1)_0$

$n=6$

$(y^{(6+1)})_0 = (y^6)_0 (6+1)$

$(y^7)_0 = 7(y^6) = 7 \times 720(y^1)_0 = 5040(y^1)_0$

Using Taylor's Maclaurin theory

$y = (y^{(0)})_0 + x(y^1)_0 + x^2 \frac{(y^2)_0}{2!} + x^3 \frac{(y^3)_0}{3!} + x^4 \frac{(y^4)_0}{4!} + x^5 \frac{(y^5)_0}{5!} + x^6 \frac{(y^6)_0}{6!} + x^7 \frac{(y^7)_0}{7!}$

$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^{(4)})_0 + \frac{x^5}{5!}(y^{(5)})_0 + \frac{x^6}{6!}(y^{(6)})_0 + \cdots$

$\cdots + \frac{x^9}{9!}(y^{(9)})_0$

$y = (y^{(0)})_0 + (x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)(y^1)$

$y = 0.0005 + 0.0005(x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$

When $x = 5$

$y = 0.0005 + 0.0005(5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)$

$= 97.8281$ m

$x = 8$

$y = 0.0005 + 0.0005(8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$

$= 1198.3745$ m

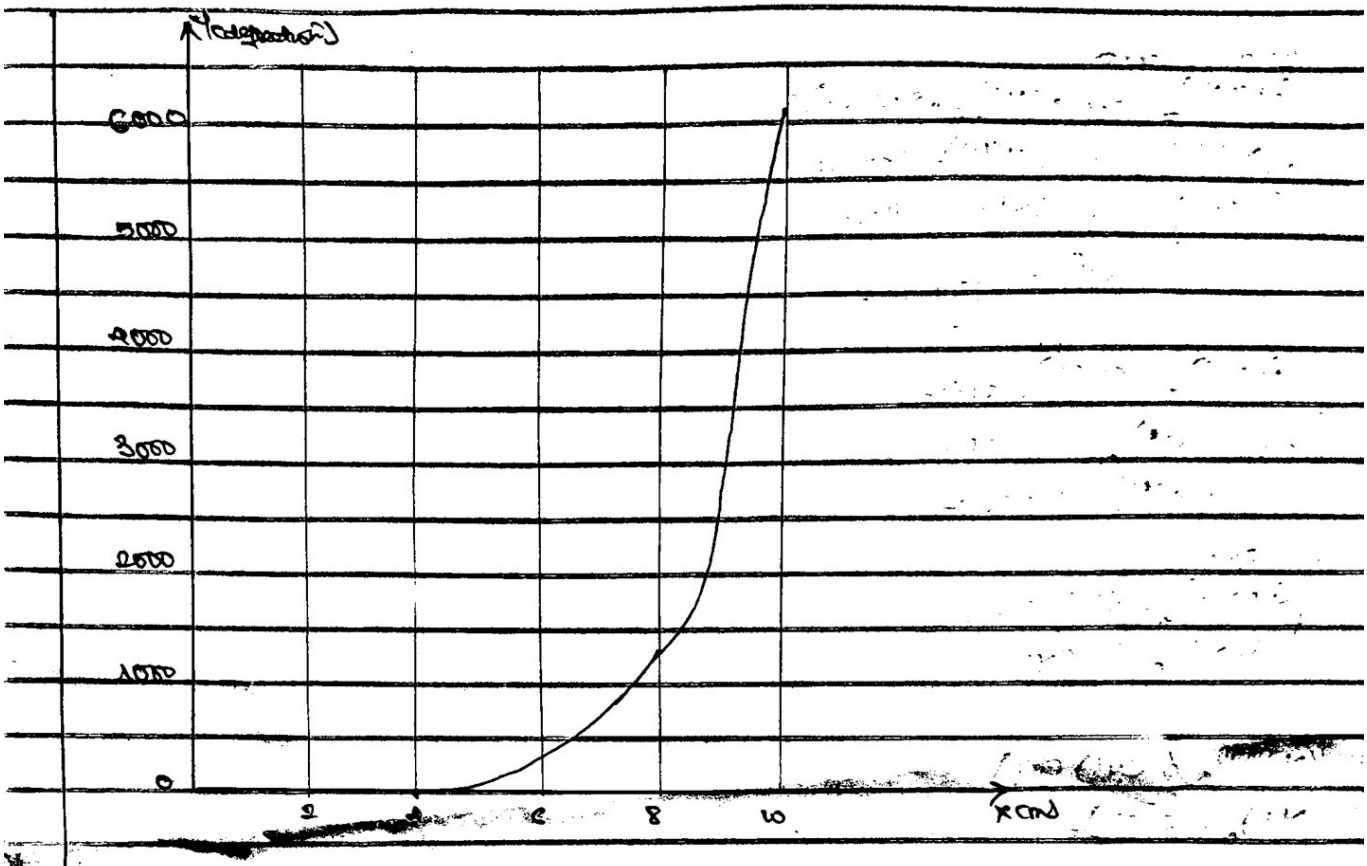
$x = 10$

$y = 0.0005 + 0.0005(10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$

$= 5555.5555$ m

Matlab file :

```
Command window
clear
cc
close all
x = 0: 0.01: 10
y = (0.0005 * (1+x)) + ((x12 + x13 + x14 + x15 + x16 + x17) * 0.0005)
yn = fliplr(y)
plot(x, yn)
xlabel('m')
ylabel('Deflection')
axis on
grid on
grid minor
```


6000
5000
4000
3000
2000
1000
0
2
4
6
8
10
x (cm)