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$x(x-1)y'' + (3x-1)y' + y = 0$

Let

$W_1 = x(x-1)y''$

$W_2 = (3x-1)y'$

$W_3 = y$

for W_1

$u = y^2$ $\quad V = x(x-1) = x^2 - x$

$u^n = y^{n+2}$ $\quad V' = 2x - 1$

$u^{(n)} = y^{n+1}$ $\quad V^{(n)} = 2$

$u^{n-2} = y^n$ $\quad V^{(n)} = 0$

for W_2

$u = y'$ $\quad V = 3x - 1$

$u^n = y^{n+1}$ $\quad V' = 3$

$u^{n-1} = y^n$ $\quad V'' = 0$

For W_3

$u = y$ $\quad V = 1$

$u^n = y^n$ $\quad V' = 0$

Recall

$$y = uV + nU^{(n-1)}V' + \frac{n(n-1)}{2!}U^{(n-2)}V^2 + \frac{n(n-1)(n-2)}{3!}U^{(n-3)}V^3 + \cdots$$

$$y = y^{(n+2)}\cdot(x^2 - x) + n(y^{(n+1)})(2x-1) + \frac{n(n-1)}{2!}y^n \cdot 2 + \frac{n(n-1)(n-2)}{3!}(y^{(n-1)})\cdot 0 + (y^{(n+1)})(3x-1)$$

$$+ ny^n \cdot 3 + \frac{n(n-1)}{2!}(y^{(n-1)})\cdot 0 + y^n \cdot 1 + n\,y^{(n-1)}\cdot 0$$

$$= (x^2 - x)(y^{(n+2)}) + n(2x-1)(y^{(n+1)}) + n(n-1)(y^{(n)}) + (3x-1)(y^{(n+1)}) + n\,3(y^n) + (y^n)$$

$$(x^2 - x)(y^{(n+2)}) + (y^{(n+1)})\cdot(n(2x-1) + (3x-1)) + (y^{(n)})\cdot(n(n-1) + 3n + 1)$$

$$= (x^2 - x)\cdot(y^{(n+2)}) + (y^{(n+1)})\cdot(2nx - n + 3x - 1) + (y^n)\cdot(n^2 - n + 3n + 1)$$

assuming $x=0$

$(y^{(n+2)})_0(-n-0)+(y^n)(n^0+2n+1)=0$

$-(y^{(n+0)})_{(n+1)}+(y^{(n)})_0(n^0+2n+1)=0$

$(y^{(n+2)})_0-(n+1)=(y^{(n)})_0(n^0+2n+1)$

$(y^{(n+2)})_0=(y^{(n)})_0\frac{(n+1)(n+1)}{n+1}$

$(y^{(n+2)})_0=(y^{(n)})_0(n+1)$

Recall $(y^0)_0=0.0005$

$(y^{(0)})_0=0.0005$

$n=0$

$(y^{(0+1)})_0=(y^0)(0+1)$

$(y^{(1)})_0=1(y^0)_0$

$n=1$

$(y^{(1+1)})_0=(y^1)(1+1)$

$(y^2)_0=2(y^1)_0$

$n=2$

$(y^{(2+1)})_0=(y^2)(2+1)$

$(y^3)_0=3y^2=3\times[y^0]=6y^0$

$n=3$

$(y^{(3+1)})_0=(y^3)(3+1)$

$(y^4)_0=4(y^3)=4\times6(y^1)=24(y^1)_0$

$n=4$

$(y^{(4+1)})_0=(y^4)(4+1)$

$(y^5)_0=5(y^4)=5\times24(y^1)_0=120(y^1)_0$

$n=5$

$(y^{(5+1)})_0=(y^5)(5+1)$

$(y^6)_0=6(y^5)=6\times120(y^1)=720(y^1)_0$

$n=6$

$(y^{(6+1)})_0=(y^6)(6+1)$

$(y^7)_0=7(y^6)=7\times720(y^1)_0=5040(y^1)_0$

Using Taylor's Maclaurin theory

$y=(y^{(0)})_0+x(y^1)_0+\frac{x^2}{2!}(y^2)_0+\frac{x^3}{3!}(y^3)_0+\frac{x^4}{4!}(y^4)_0+\frac{x^5}{5!}(y^5)_0+\frac{x^6}{6!}(y^6)_0+\frac{x^7}{7!}(y^7)_0$

$$y = (y^{(0)})_0 + x(y^{(0)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^{(4)})_0 + \frac{x^5}{5!}(y^{(5)})_0 + \frac{x^6}{6!}(y^{(6)})_0 + \cdots + \frac{x^n}{n!}(y^{(n)})_0$$

$$y = y^{(0)}_0 (1+x) + (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)(y^1)$$

$$y = 0.0005 (1+x) + 0.0005 (x^2 + x^3 + x^4 + x^5 + x^6 + x^7)$$

When $x = 5$

$$y = 0.0005(1+5) + 0.0005 (5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7)$$

$$= 11.828 \text{ m}$$

$x = 8$

$$y = 0.0005 (1+8) + 0.0005 (8^2 + 8^3 + 8^4 + 8^5 + 8^6 + 8^7)$$

$$= 11\ 98.3285 \text{ m}$$

$x = 10$

$$y = 0.0005 (1+10) + 0.0005 (10^2 + 10^3 + 10^4 + 10^5 + 10^6 + 10^7)$$

$$= 5555.5555 \text{ m}$$

Practical file

Command window

clear

cc

close all

x = 0 : 0.01 : 10

y = (0.0005 * (1+x)) + (x12 + x13 + x14 + x15 + x16 + x17) * 0.0005)

yn = fliplr(y)

Plot (x, yn)

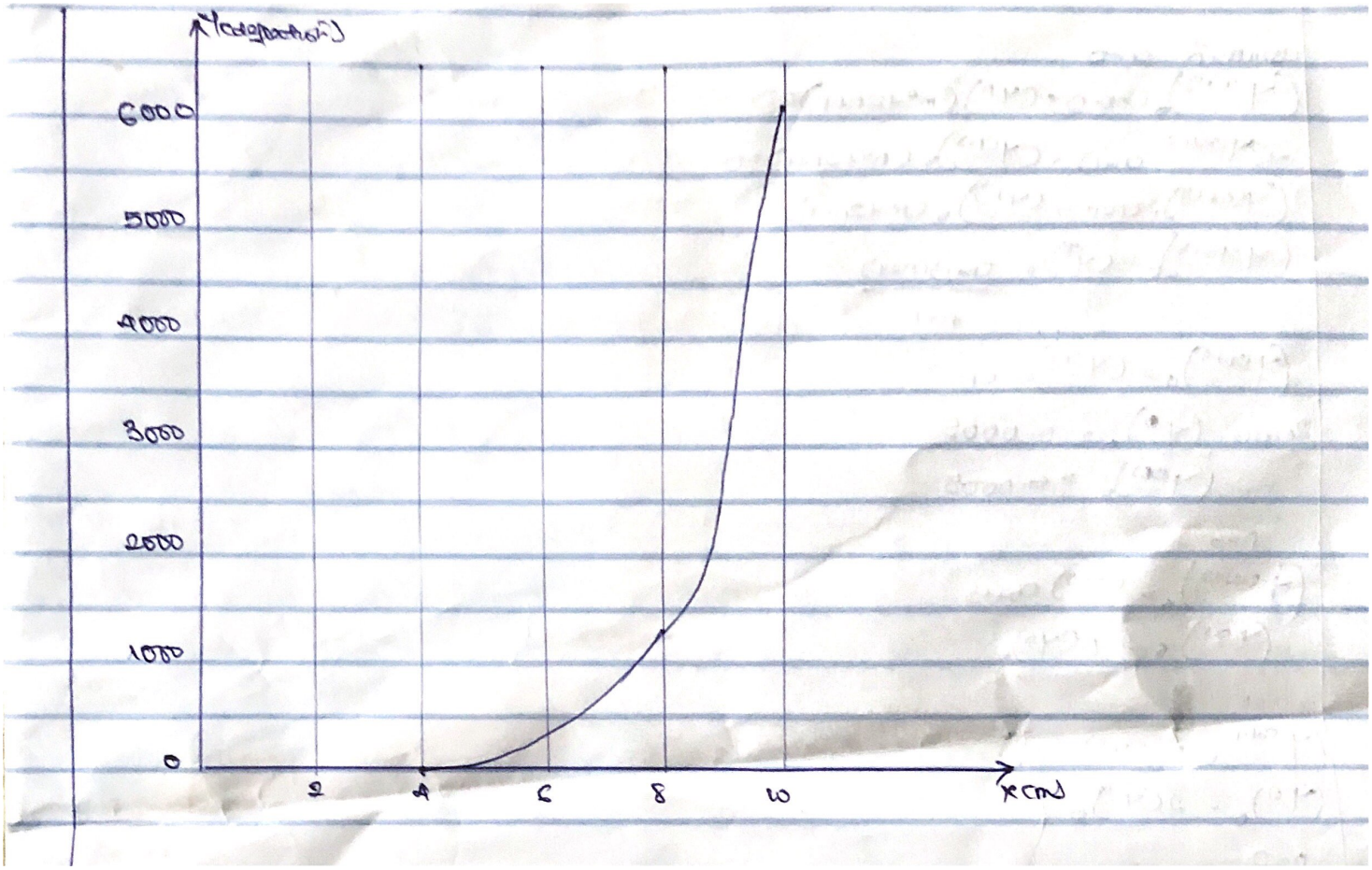
xlabel ('m')

ylabel ('Deflection')

axis on

grid on

grid minor


Y (Acceleration)
6000
5000
4000
3000
2000
1000
0
2
4
6
8
10
X (cm)