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1.

$y = 3e^{2x}$ and $y = 3e^{-x}$ at $x=1$ and $x=2$

Soln

$y = 3e^{2x}$ $\quad y = 3e^{-x}$ $\quad \Rightarrow$ Let $f(x) = 3e^{2x}$ and $g(x) = 3e^{-x}$

For two Curves $= \int (f(x) - g(x))$

$\therefore \int (3e^{2x} - 3e^{-x}) \Rightarrow$

$= \int (3(e^{2x} - e^{-x}))\, dx$

$= 3\int e^{2x} - e^{-x}\, dx$

$= 3\left[\frac{e^{2x}}{2} + e^{-x}\right]_1^2 dx$

$= 3\left(\left[\frac{e^{2(2)}}{2} + e^{-2}\right] - \left[\frac{e^{2(1)}}{2} + e^{-1}\right]\right) dx$

$= 3\left(\left(\frac{e^4}{2} + e^{-2}\right) - \left(\frac{e^2}{2} + e^{-1}\right)\right) dx$

$3\left(\left(\frac{54.6}{2} + 0.1353\right) - \left(\frac{7.3890}{2} + 0.3679\right)\right)$

$= 3(27.4344 - 4.06241)$

$3(23.3720)$

$= 70.12 \text{ unit}^2$

2

$y = 2\sin\frac{\pi}{10}t \qquad x = 2 + 2t - 2\cos\frac{\pi}{10}t$

Soln

$y = \int 2\sin\frac{\pi}{10}t\, dx$

$x = 2 + 2t - 2\cos\frac{\pi}{10}t$

$\frac{\delta x}{\delta t} = 2 + \frac{2\pi}{10}\sin\frac{\pi}{10}t$

$\therefore \delta x = 2 + \frac{2\pi}{10}\sin\frac{\pi}{10}t\ \delta t$

$y = \int 2\sin\frac{\pi}{10}t\left(2 + \frac{2\pi}{10}\sin\frac{\pi}{10}t\right)\delta t$

$y = \int 4\sin\frac{\pi}{10}t + \frac{4\pi}{10}\sin^2\frac{\pi}{10}t\ \delta t$

$y = 4\int \sin\frac{\pi}{10}t + \frac{\pi}{10}\sin^2\frac{\pi}{10}t\ \delta t$

From $\cos 2A = \cos^2 A - \sin^2 A$ --- i

Substituting $\cos 2\frac{\pi}{10}t = \cos^2\frac{\pi}{10}t - \sin^2\frac{\pi}{10}t$ --- ii

~~Cos²~~

Rem; $\cos^2\frac{\pi}{10}t + \sin^2\frac{\pi}{10}t = 1$

$\therefore \cos^2\frac{\pi}{10}t = 1 - \sin^2\frac{\pi}{10}t$ --- iii

Putting eqn iii into eqn ii

$\therefore \cos\frac{2\pi}{10}t = 1 - \sin^2\frac{\pi}{10}t - \sin^2\frac{\pi}{10}t$

$\cos\frac{2\pi}{10}t = 1 - 2\sin^2\frac{\pi}{10}t$

$2\sin^2\frac{\pi}{10}t = 1 - \cos\frac{2\pi}{10}t$

$\sin^2\frac{\pi}{10}t = \frac{1}{2}\left(1 - \cos\frac{2\pi}{10}t\right)$

$\therefore y = 4\int_0^{10}\sin\frac{\pi}{10}t + \frac{\pi}{10}\left(\frac{1}{2}\left(1 - \cos\frac{2\pi}{10}t\right)\right)\delta t$

$y = 4\int_0^{10}\sin\frac{\pi}{10}t + \frac{\pi}{20}\left(1 - \cos\frac{2\pi}{10}t\right)\delta t$

$y = 4\left[-\frac{\cos\frac{\pi}{10}t}{\pi/10} + \frac{\pi}{20}\left(t - \frac{\sin\frac{\pi}{5}t}{\pi/5}\right)\right]_0^{10}$

$$y = 4\left[\frac{10}{\pi}\left(-\cos\frac{\pi}{10}t\right) + \frac{\pi}{20}\left(t - \left(\frac{5}{\pi}\left(\sin\frac{\pi}{5}t\right)\right)\right)\right]_0^{10}$$

$$y = 4\left[\left[\frac{10}{\pi}\left(-\cos\frac{\pi}{10}\cdot 10\right) + \frac{\pi}{20}\left(10 - \left(\frac{5}{\pi}\left(\sin\frac{\pi}{5}\cdot 10\right)\right)\right)\right] - \left[\frac{10}{\pi}\left(-\cos\frac{\pi}{10}\cdot 0\right)\right.\right.$$

$$\left.\left. + \frac{\pi}{20}\left(0 - \left(\frac{5}{\pi}\left(\sin\frac{\pi}{5}\cdot 0\right)\right)\right)\right]\right]$$

$$= 4\left[\left[-\frac{10}{\pi}\cos\pi + \frac{\pi}{20}\left(10 - \frac{5}{\pi}\sin\pi\right)\right] - \left[-\frac{10}{\pi}\cdot\cos 0 + \frac{\pi}{20}\left(0 - \frac{5}{\pi}\sin 0\right)\right]\right]$$

$$4\left[\left[\left(-\frac{10}{\pi}\cdot -1\right) + \frac{\pi}{20}(10 - 0)\right] - \left[\left(-\frac{10}{\pi}\cdot 1\right) + \frac{\pi}{20}(0)\right]\right]$$

$$4\left(\left[10/\pi + \pi/2\right] - \left[-10/\pi + 0\right]\right)$$

$$= 4\left(\frac{20+\pi^2}{2\pi} + \frac{10}{\pi}\right)$$

$$= 4\left(\frac{10}{\pi} + \frac{\pi}{2} + \frac{10}{\pi}\right)$$

$$= 4\left(\frac{20}{\pi} + \frac{\pi}{2}\right)$$

$$= \frac{80}{\pi} + \frac{4\pi}{2}$$

$$= \frac{80}{\pi} + 2\pi$$

$$= 31.745 \text{ unit}^2$$