

$$x(x-1)y'' + (3x-1)y' + y = 0$$

Part a

$$x(x-1)y'' , u = y'' , u^n = y^{n+2}$$

$$v = 2x^2 - x , dv = 2x - 1 , v'' = 2$$

$$y' = u'v + nu^{n-1}v' + \frac{n(n-1)u^{n-2}u''v^2}{2!}$$

$$= x(x-1)y^{n+2} + ny^{n+1}(2x-1) + \frac{n(n-1)y^{n+2}}{2}$$

$$= x(x-1)y^{n+2} + 2ny^{n+1}(x-1) + \frac{(n^2-n)}{2}y^{n+2}$$

Part b

$$(3x-1)y' , u = y' , u^n = y^{n+1}$$

$$v = 3x - 1 , v' = 3$$

$$y^n = u^n v + nu^{n-1}v'$$

$$= (3x-1)y^{n+1} + n3y^n$$

Part c

$$y , u = y , u^n = y^n$$

$$\therefore \text{Part A} + \text{Part B} + \text{Part C} = 0$$

$$x(x-1)y^{n+2} + ny^{n+1}(2x-1) + \frac{(n^2-n)}{2}y^{n+2} + (3x-1)y^{n+1} + 3ny^n$$

$$\therefore x(x-1)y^{n+2} + (2x^2 + 2x - 1)y^{n+1} + (n^2 + 2n + 1)y^n = 0$$

$$\text{when } n=0$$

$$(2x^2 + 2x - 1)y^{n+1} + (n^2 + 2n + 1)y^n = 0$$

$$- (n^2 + 2n + 1)y^{n+1} = - [ (n+1)(n+1) ] y^n = 0$$

$$\therefore y^{n+1} = (n+1)y^n$$

$$\text{when } n=0$$

$$y_0^1 = y_0$$

$$\text{when } n=1$$

$$y_0^2 = 2y_0^1 , (2y_0)$$

$$\text{when } n=2$$

$$y_0^3 = 3y_0''', \quad 3(2y_0''') = 6y_0''' \\ \text{when } n=3$$

$$y_0^4 = 4y_0^{(4)} + (6y_0''') = 24y_0^{(4)} \\ \text{when } n=4$$

$$y_0^5 = 5y_0^{(5)} + 5(24y_0^{(4)}) = 120y_0^{(5)} \\ \text{when } n=5$$

$$y_0^6 = 6y_0^{(6)} + 6(120y_0^{(5)}) = 720y_0^{(6)} \\ \text{when } n=6$$

$$y_0^7 = 7y_0^{(7)} + 7(720y_0^{(6)}) = 5040y_0^{(7)} \\ \text{when } n=7$$

From Maclaurin Series

$$y = \frac{x^0 y_0}{0!} + \frac{x^1 y_0'}{1!} + \frac{x^2 y_0''}{2!} + \frac{x^3 y_0'''}{3!} + \frac{x^4 y_0^{(4)}}{4!} + \frac{x^5 y_0^{(5)}}{5!} + \frac{x^6 y_0^{(6)}}{6!} \\ + \frac{x^7 y_0^{(7)}}{7!}$$

$$y = y_0 + xy_0' + \frac{x^2 y_0''}{2!} + \frac{x^3 y_0'''}{3!} + \frac{x^4 \cdot 24y_0^{(4)}}{4!} + \frac{x^5 \cdot 120y_0^{(5)}}{5!} + \frac{x^6 \cdot 720y_0^{(6)}}{6!} \\ + \frac{x^7 \cdot 5040y_0^{(7)}}{7!}$$

$$y = y_0 + xy_0' + x^2 y_0'' + x^3 y_0''' + x^4 y_0^{(4)} + x^5 y_0^{(5)} + x^6 y_0^{(6)} + x^7 y_0^{(7)}$$

$$y = y_0 [1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \dots]$$

$$\text{where } y_0 = 0.0005 \quad \frac{1}{2} y_0' = 0.0005$$

$$\text{where } y_0 = y_0'$$

b) when  $x = 5u$

$$y = 0.0005 [1 + 5 + 25 + 125 + 625 + 3125 + 15625 + 78125]$$

$$y = 0.0005 [92656]$$

$$y = 46.328$$

$$y = [11, 11, 11] \cdot 0.0005$$

$$y = 5,555.355$$

c) Command window

clear

clc

Syms x

Syms y

$$a = (0.40)^x;$$

$$y = 0.0005 (1 + 2x + (ax)^c) + (2e^{-x/5}) + (2e^{-x/4}) + (x.^{1.5}) + (x.^{1.6}) + (x.^{1.7});$$

Plot (x,y)

grid on

grid minor

x label ('x')

y label ('structural reformations')